COURSE NAME: THEORITICAL MECHANICS

COUSRE CODE: MA 278

NAME : Webgee

INDEX NUMBER: xxxxxxx

1)

a) Define moment of inertia. Show that the moment of inertia of a uniform hollow cylinder of inner radius R_1, outer radius R_2 and mass M, is $I = \frac{1}{2}(R_1^2 + R_2^2)$ as in figure 1, if the rotation axis is through the center along the axis of symmetry.

b) Obtain the moment of inertia for a solid cylinder

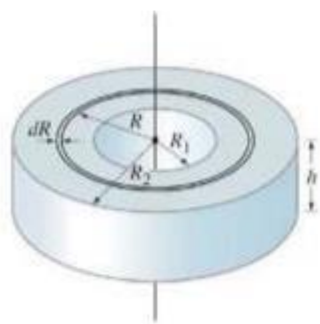

Figure 1

Soln

a) Moment of inertia: **Moment of inertia** (I), also called "**angular mass**" (kg·m^2), is the inertia of a rotating body with respect to its rotation.
It is a rotating body's resistance to angular acceleration or deceleration, equal to the product of the mass and the square of its perpendicular distance from the axis of rotation

The moment of inertia of a thin ring of radius R is mR^2.So we divide the cylinder into thin concentric cylindrical rings or hoops of thickness dR. If the density (mass per unit volume) is ρ, then

$$dm = \rho\, dV,$$

where dV is the volume of the thin ring of radius R, thickness dR, and height h. Since $dV = (2\pi R)(dR)(h)$, we have

$$dm = 2\pi\rho h R\, dR.$$

The moment of inertia is obtained by integrating (summing) over all these rings:

$$I = \int R^2\, dm$$

$$\Longrightarrow I = \int R^2 \cdot 2\pi\rho h R\, dR$$

$$I = \int_{R_1}^{R_2} 2\pi\rho h\, R^3 dR$$

$$I = 2\pi\rho h \left[\frac{R^4}{4}\right]_{R_1}^{R_2}$$

$$I = 2\pi\rho h \left(\frac{R_2^4 - R_1^4}{4}\right)$$

$$= \pi\rho h \left(\frac{R_2^4 - R_1^4}{2}\right)$$

where we are given that the cylinder has uniform density, ρ = constant. (If this were not so, we would have to know ρ as a function of R before the integration could be carried out.) The volume V of this hollow cylinder is

$V = (\pi R_2^2 - \pi R_1^2)h$, so its mass M is

$M = \rho V = \rho(\pi R_2^2 - \pi R_1^2)h$

Since $(R_2^4 - R_1^4) = (R_2^2 - R_1^2)(R_2^2 + R_1^2)$,

We have

$$I = \frac{\pi \rho h}{2}(R_2^2 - R_1^2)(R_2^2 + R_1^2)$$

$$= \frac{1}{2} M(R_2^2 + R_1^2)$$

As required.

a) For a solid cylinder, $R_1 = 0$ and if we set $R_2 = R_0$,then

$$= \frac{1}{2} MR_0^2$$

2)

a) Can a small force ever exert a greater torque than a larger force? Explain.
b) If one object has a greater speed than a second object, does the first necessarily have a greater acceleration? Explain.
c) Can an object be increasing in speed as its acceleration decreases? If so, give an example. If not, explain.

Solution

a) Yes, a small force can exert a greater torque than large force. Torque equals the product of the and the lever arm distance. A force of 5.0 N with a lever arm distance of 3.0 m produces a torque of 15 N m. A 10 N force with a lever arm distance of 1.0 m produces a torque of 10 N m. therefore, if the lever arm distance is large enough, the torque exerted by small force will be than that exerted by a large force with a small lever arm distance.

b) No, if one object has a greater speed than a second object, it does not necessarily have a greater acceleration. For example, consider a speeding car, traveling at constant velocity, which passes a stopped police car. The police car will accelerate from rest to try to catch the speeder. The speeding car has a greater speed than the police car (at least initially!), but has zero acceleration. The police car will have an initial speed of zero, but a large acceleration.

c) Yes, an object can be increasing in speed as the acceleration decreases. To understand the same, you must get the idea of the meaning of acceleration. Theoretically, acceleration is the rate of change of velocity while speed is the magnitude of the velocity. Therefore, in a situation where the change in velocity of an object is less, the acceleration would be smaller relative to a situation in which the change in velocity over the same time is larger. For example, consider the motion of the car along the straight road. Assume that the car is moving initially with velocity 20 km/h and suddenly accelerates to bring the final velocity of 40km/h. The acceleration of the car is given by the change of velocity over the elapse time.

   Now, consider that the car accelerates again and move to a higher velocity of 50 km/h and does so in the same amount of time.

   Therefore, the change in velocity of the car when it moves from 40 km/h to 50 km/h over some time difference is relatively smaller in magnitude as compared to change in velocity from 20 km/h to 40 km/h over the same time difference