UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

SECOND SEMESTER EXAMINATIONS, MAY 2019

COURSE NO: MA 376

COURSE NAME: OPTIMISATION TECHNIQUES

CLASS: MA III Unihubgh.com TIME: 3 HOURS

Name: ______________________ Index Number: ______________

INTRUCTIONS: **Attempt ALL Questions.**

Identify and Circle *the choice that best completes the statement on the question booklet in this section. Section A is worth 50 marks and has a total of 25 multiple-choice questions. Each correct answer is worth 2 points.*

SECTION A

1. The function $f(x)=x^2$ is
   (a) Unimodal and convex
   (b) Multimodal and convex
   (c) Unimodal and strictly convex
   (d) Unimodal and concave

2. The function $f(x)=|x|$ is
   (a) Concave
   (b) Strictly concave
   (c) Convex
   (d) Strictly convex

3. For the maximization of the function $f(x)=6.64+1.2x-x^2$, suppose the starting search interval using Golden Section Search is $[0,1]$, how small is the search interval after 5 iterations?
   (a) 0.056
   (b) 0.090
   (c) 0.008
   (d) Cannot be determined in advance

4. At the k^{th} stage of the Davies, Swann and Campey's (D.S.C) algorithm with $x^{(k)}$ known, we compute $x^{(k+1)}$ as
   (a) $x^{(k)}+2\Delta x$
   (b) $x^{(k)}+\Delta x$
   (c) $x^{(k)}-2\Delta x$
   (d) $x^{(k)}-\Delta x$

5. Using the D.S. C, if $f(x^{(1)})>f(x^{(2)})$, where Δx is the initial step length, we set $x^{(1)}$ to be equal to
   (a) $x^{(0)}-\Delta x$
   (b) $x^{(0)}-2\Delta x$
   (c) $x^{(1)}-\Delta x$
   (d) $x^{(0)}+\Delta x$

6. Find the minimum value of the function $f(x)=x^4-2x^2+1$
   (a) 2
   (b) 0
   (c) 1
   (d) -1

Use the information below to answer ***Questions 7 and 8****.*

Using three-point equal search algorithm,

7. Its efficiency is

(a) $\frac{1}{2^{(m-1)/2}}$ (c) $\frac{1}{2^{m-1}}$

(b) $\frac{1}{2^{(m)/2}}$ (d) $\frac{1}{2^{m-2}}$

8. The length of the interval of uncertainty after n applications is

(a) $\frac{b-a}{2^{n-2}}$ (c) $\frac{b-a}{2^{n}}$

(b) $\frac{b-a}{2^{n-1}}$ (d) $\frac{b+a}{2^{n-1}}$

Use the information below to answer ***Questions 9 to 14****.*

Using the Fibonacci search for the minimum value of the function $f(x)=\frac{(x^6-x+1)}{e^{x^2+1}}$ *in the interval* $-1\le x\le 1$, *reducing the interval of uncertainty to less than 2 % (using the usual notations).*

9. How many evaluations are required to solve this problem?

(a) 19 (b) 14 (c) 9 (d) 6

10. Evaluate $x_{u,1}$

(a) -0.236364 (b) 0.25 (c) 0.50 (d) 0.236346

11. Evaluate $f(x_{l,1})$

(a) 0.265722 (b) 0.43018 (c) -0.265722 (d) -0.43018

12. Evaluate $f(x_{u,1})$

(a) 0.265722 (b) 0.43018 (c) -0.265722 (d) 0.5618

13. Evaluate I_2

(a) 0. 472728 (b) 1.472728 (c) 0.7472 (d) 1.23636

14. Evaluate $x_{u,2}$

(a) 0.617647 (b) -0.5227 (c) -0.52727 (d) 0.52727

15. The quadratic formula use by DSC method to compute the minimum point is given by

(a) $\tilde{x}^{*}=x^{(b)}+\frac{\left\{f\left(x^{(a)}\right)+f\left(x^{(b)}\right)\right\}\times h}{2\left\{f\left(x^{(a)}\right)+2f\left(x^{(b)}\right)+f\left(x^{(c)}\right)\right\}}$

(b) $\tilde{x}^{*}=x^{(b)}-\frac{\left\{f\left(x^{(a)}\right)-f\left(x^{(b)}\right)\right\}\times h}{2\left\{f\left(x^{(a)}\right)-2f\left(x^{(b)}\right)+f\left(x^{(c)}\right)\right\}}$

(c) $\bar{x}^{*} = x^{(b)} + \dfrac{\left\{f\left(x^{(a)}\right) - f\left(x^{(b)}\right)\right\} \times h}{2\left\{f\left(x^{(a)}\right) - 2f\left(x^{(b)}\right) - f\left(x^{(c)}\right)\right\}}$

(d) $\bar{x}^{*} = x^{(b)} + \dfrac{\left\{f\left(x^{(a)}\right) - f\left(x^{(b)}\right)\right\} \times h}{2\left\{f\left(x^{(a)}\right) - 2f\left(x^{(b)}\right) + f\left(x^{(c)}\right)\right\}}$

16. The efficiency of the dichotomous search in the interval $[a, b]$ is given by the expression

(a) $\dfrac{\dfrac{b-a}{2^{\frac{n}{2}}} + \dfrac{\dfrac{\varepsilon}{2}\left(1-\left(\dfrac{\varepsilon}{2}\right)^{\frac{n}{2}}\right)}{1-\dfrac{\varepsilon}{2}}}{b-a}$

(b) $\dfrac{b-a}{2^{n/2}} + \dfrac{\varepsilon}{2}\left(1-\left(\dfrac{\varepsilon}{2}\right)^{\frac{n}{2}}\right)$

(c) $\dfrac{b-a}{2^{\frac{n+1}{2}}} + \dfrac{\varepsilon}{2}\dfrac{\left(1-\left(\dfrac{\varepsilon}{2}\right)^{\frac{n-1}{2}}\right)}{1-\dfrac{\varepsilon}{2}}$

(d) $\dfrac{b-a}{2^{\frac{n}{2}}} + \dfrac{\varepsilon}{2}\dfrac{\left(1-\left(\dfrac{\varepsilon}{2}\right)^{\frac{n}{2}}\right)}{1-\dfrac{\varepsilon}{2}}$

17. The minimum value of the function $f(x) = \begin{cases} x-1 & \text{if } x-1 \geq 0 \\ -(x-1) & \text{if } x-1 < 0 \end{cases}$ is

(a) 0 (b) 1 (c) -1 (d) 2

18. At the k^{th} stage of the Powell's algorithm with an initial guess $x^{(k)}$ known, we compute $x^{(k+1)}$ as

(a) $x^{(k)} + 2\Delta x$ (b) $x^{(k)} + \Delta x$ (c) $x^{(k)} - 2\Delta x$ (d) $x^{(k)} - \Delta x$

19. After a second application of two-point equal interval search algorithm the length of the new interval is

(a) $\left(\dfrac{2}{3}\right)^2 (b-a)$

(b) $\left(\dfrac{2}{3}\right)^2 (b+a)$

(c) $\left(\dfrac{1}{2}\right)^2 (b+a)$

(d) $\left(\dfrac{1}{2}\right)^2 (b-a)$

20. In three-point equal search we evaluate three functional values in the interval $[a, b]$ as

(a) $a - \dfrac{(b-a)}{4}, a - \dfrac{2(b-a)}{4}$ and $a - \dfrac{3(b-a)}{4}$

(b) $a + \dfrac{(b+a)}{4}, a + \dfrac{2(b+a)}{4}$ and $a + \dfrac{3(b+a)}{4}$

(c) $a + \dfrac{(b-a)}{4}, a + \dfrac{2(b-a)}{4}$ and $a + \dfrac{3(b-a)}{4}$

(d) $a - \dfrac{(b+a)}{4}, a - \dfrac{2(b+a)}{4}$ and $a - \dfrac{3(b+a)}{4}$

Use the information below to answer ***Questions 21 to 24.***

Using Powell's algorithm to minimize $f(\lambda) = \lambda^4 - 20\lambda^3 + 0.1\lambda$ *with* $\varepsilon = 0.001,\ \lambda^{(1)} = 0.0$ *and* $\Delta\lambda = 0.5$

21. Find $f\left(\lambda^{(2)}\right)$

    (a) 0.5000
    (b) -2.3875
    (c) 1.0000
    (d) None of the above

22. Find $f\left(\lambda^{(3)}\right)$

    (a) -18.9000
    (b) 2.3375
    (c) -2.3875
    (d) None of the above

23. Find $\tilde{\lambda}^*$

    (a) -0. 1655
    (b) 0.1655
    (c) 18.9000
    (d) None of the above

24. Find $f\left(\tilde{\lambda}^*\right)$

    (a) 0.0734
    (b) 0.1655
    (c) -0.0734
    (d) None of the above

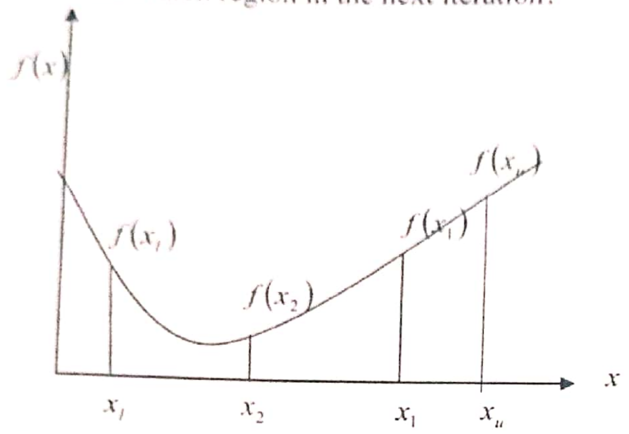
25. If the Golden Section rule is applied in a search, the interval of uncertainty is reduced by the value of

    (a) -1.618
    (b) 0.618
    (c) 1.618
    (d) -0.618

D. D.S.C is a simultaneous search method

E. The efficiency of any search method is computed by $\frac{\Delta^{(n)}}{\Delta^{(0)}}$, where $\Delta^{(n)}$ is the interval of uncertainty after the n functional evaluations and $\Delta^{(0)}$ the original interval of uncertainty.

28. Consider the problem of finding the minimum of the function shown below. Given the intermediate points in the drawing, what would be the search region in the next iteration?



A. $[x_2, x_u]$ B. $[x_1, x_u]$ C. $[x_l, x_1]$ D. $[x_l, x_2]$ E. $[x_1, x_2]$

Use the information below to answer Questions **29** to **34**.

Using the Fibonacci search for the minimum value of the function $f(x)=\frac{(x^6-x+1)}{e^{x^2+1}}$ in the interval $-1 \le x \le 1$, reducing the interval of uncertainty to less than 2 % (using the usual notations),

29. How many evaluations are required to solve this problem?
A. 24 B. 19 C. 14 D. 9 E. 6

30. Evaluate $x_{u,1}$ A. -0.236364 B. 0.25 C. 0.50 D. 0.743 E. 0.236346
31. Evaluate $f(x_{l,1})$ A. 0.265722 B. 0.43018 C. -0.265722 D. -0.43018 E. 0.5618
32. Evaluate $f(x_{u,1})$ A. 0.265722 B. 0.43018 C. -0.265722 D. 0.5618 E. -0.5618
33. Evaluate I_2 A. 0. 472728 B. 1.472728 C. 0.7472 D. 0.7878 E. 1.23636
34. Evaluate $x_{u,2}$ A. 0.617647 B. -0.5227 C. -0.52727 D. 0.52727 E. 0.431469

Use the information below to answer Questions **35** to **38**.

Using Powell's algorithm to minimize $f(x)=x^4-4x-1, x \in \Re$, with TOL=$\varepsilon=0.001$, $x^{(1)}=0.0$ and $\Delta x=0.5$

35. Find $f(x^{(2)})$ A. -1 B. -2.9375 C. -4 D. -3.078 E. -3.35
36. Find $f(x^{(3)})$ A. -1 B. -2.9375 C. -4 D. -3.078 E. -3.35
37. Find $\tilde{x}^*$ A. -1.25 B. -1.05 C. -1.35 D. 1.35 E. 0.5
38. Find $f(\tilde{x}^*)$ A. 6.44 B. 4.42 C. 7.72 D. -3.08 E. -2.94

Examiner: Dr E. N. Wiah

Section B is also worth 50 marks. It has 3 numerical problems which require more extensive calculations. In order to obtain full credit for any question, all work must be shown in detail. Full credit will not be given for an answer not supported by working.

SECTION B

- Leave your results in 4 decimal Places

1. (a) Define convex function? Hence examine whether the following function is convex or concave and find the points of optima:

$$f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3$$

(b) Compute the Gradient and Hessian if $f_1 = \ln\left(e^{x_1} + e^{x_2} + e^{x_3}\right)$, $f_2 = e^{x_3}\sin(x_1)\sin(x_2)$ and $f_3 = \ln(2 + \sin(x_1) + \cos(x_2))$

2. Find the minimum of the function $f(\lambda) = \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left(1 - \frac{0.5}{1+\lambda^2}\right) + \lambda$ using the dichotomous search method in the interval $[0, 3]$ using a value of $\delta = 0.0001$. Stop when the interval of uncertainty is less than 5% of the original.

3. Minimize $f(x) = 0.65 - \frac{0.75}{1+x^2} - 0.65x\tan^{-1}\frac{1}{x}$ with a step size of 0.1 starting from point 0.0 using the Coggin's algorithm. Stop when $|\bar{x}_{k+1} - \bar{x}_k| < 0.01$.

Examiners: Dr E. N. Wiah / Dr H. Otoo

Question 1

(a) (i) Convex function.

A real-valued function, f, defined on an interval is called convex if for any two points say X and y in its domain C and any time t in [0, 1], we have

$$f(tx+(1-t)y) \leq tf(x) + (1-t)f(y).$$

(ii) $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 6x_1x_3 - 5x_2x_3$

Gradient of f, $\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \frac{\partial f}{\partial x_3} \end{bmatrix} = 0$

$$\frac{\partial f}{\partial x_1} = 2x_1 - 4x_2 + 6x_3$$

$$\frac{\partial f}{\partial x_2} = 4x_2 - 4x_1 - 5x_3$$

$$\frac{\partial f}{\partial x_3} = -14x_3 + 6x_1 - 5x_2$$

$$\Rightarrow \nabla f = \begin{bmatrix} 2x_1 - 4x_2 + 6x_3 \\ -4x_1 + 4x_2 - 5x_3 \\ 6x_1 - 5x_2 - 14x_3 \end{bmatrix} = 0$$

We have,

$2x_1 - 4x_2 + 6x_3 = 0$ ------ (1)

$-4x_1 + 4x_2 - 5x_3 = 0$ ---- (2)

$6x_1 - 5x_2 - 14x_3 = 0$ ------(3)

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Solving equations (1), (2) and (3) Simultaneously gives
$x_1 = 0$, $x_2 = 0$ and $x_3 = 0$
$\Rightarrow (x_1, x_2, x_3) = (0, 0, 0)$

$\therefore$ Computing the Hessian matrix of f

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix}$$

$$\frac{\partial^2 f}{\partial x_1^2} = 2, \quad \frac{\partial^2 f}{\partial x_1 \partial x_2} = -4, \quad \frac{\partial^2 f}{\partial x_1 \partial x_3} = 6$$

$$\frac{\partial^2 f}{\partial x_2 \partial x_1} = -4, \quad \frac{\partial^2 f}{\partial x_2^2} = 4, \quad \frac{\partial^2 f}{\partial x_2 \partial x_3} = -5$$

$$\frac{\partial^2 f}{\partial x_3 \partial x_1} = 6 \quad \frac{\partial^2 f}{\partial x_3 \partial x_2} = -5 \quad \frac{\partial^2 f}{\partial x_3^2} = -14$$

$$\Rightarrow H = \begin{bmatrix} 2 & -4 & 6 \\ -4 & 4 & -5 \\ 6 & -5 & -14 \end{bmatrix}$$

Using $|H - \lambda I| = 0$ to compute for the eigenvalues of H

$$\Rightarrow \left| \begin{bmatrix} 2 & -4 & 6 \\ -4 & 4 & -5 \\ 6 & -5 & -14 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & -4 & 6 \\ -4 & 4-\lambda & -5 \\ 6 & -5 & -14-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\begin{vmatrix} (4-\lambda) & -5 \\ -5 & (-14-\lambda) \end{vmatrix} + 4\begin{vmatrix} -4 & -5 \\ 6 & (-14-\lambda) \end{vmatrix} + 6\begin{vmatrix} -4 & (4-\lambda) \\ 6 & -5 \end{vmatrix} = 0$$

$$(2-\lambda)[(4-\lambda)(-14-\lambda) - 25] + 4[-4(-14-\lambda)+30] + 6[20-6(4-\lambda)] = 0$$

$$(2-\lambda)[-56-4\lambda+14\lambda+\lambda^2-25] + 4[56+4\lambda+30] + 6[20-24+6\lambda] = 0$$

$$(2-\lambda)[-81+10\lambda+\lambda^2] + 4[86+4\lambda] + 6[-4+6\lambda] = 0$$

$$-162+20\lambda+2\lambda^2+81\lambda-10\lambda^2-\lambda^3+344+16\lambda-24+36\lambda=0$$

$$-\lambda^3-8\lambda^2+15\lambda+158=0 \text{ ------ (a)}$$

Solving equation (a) gives

$\lambda_1 = -16.6293$, $\lambda_2 = 9.6172$ and $\lambda_3 = -0.9879$

Since $\lambda_1 < 0$, $\lambda_1 > 0$ and $\lambda_3 < 0$, implies that the function f is neither convex nor concave. The points of optima is $(0, 0, 0)$.

(b) $f_1 = \ln(e^{x_1^2} + e^{x_2^2} + e^{x_3^2})$, $f_2 = e^{x_1}\sin(x_1)\sin(x_2)$

$f_3 = \ln(2 + \sin(x_1) + \cos(x_2))$

$$\begin{vmatrix} 2-\lambda & -4 & 6 \\ -4 & 4-\lambda & -5 \\ 6 & -5 & -14-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)\begin{vmatrix} (4-\lambda) & -5 \\ -5 & (-14-\lambda) \end{vmatrix} + 4\begin{vmatrix} -4 & -5 \\ 6 & (-14-\lambda) \end{vmatrix} + 6\begin{vmatrix} -4 & (4-\lambda) \\ 6 & -5 \end{vmatrix} = 0$$

$(2-\lambda)[(4-\lambda)(-14-\lambda) - 25] + 4[-4(-14-\lambda) + 30] + 6[20 - 6(4-\lambda)] = 0$

$(2-\lambda)[-56 - 4\lambda + 14\lambda + \lambda^2 - 25] + 4[56 + 4\lambda + 30] + 6[20 - 24 + 6\lambda] = 0$

$(2-\lambda)[-81 + 10\lambda + \lambda^2] + 4[86 + 4\lambda] + 6[-4 + 6\lambda] = 0$

$-162 + 20\lambda + 2\lambda^2 + 81\lambda - 10\lambda^2 - \lambda^3 + 344 + 16\lambda - 24 + 36\lambda = 0$

$-\lambda^3 - 8\lambda^2 + 15\lambda + 158 = 0$ ------ (a)

Solving equation (a) gives

$\lambda_1 = -16.6293$, $\lambda_2 = 9.6172$ and $\lambda_3 = -0.9879$

Since $\lambda_1 < 0$, $\lambda_1 > 0$ and $\lambda_3 < 0$, implies that the function f is neither convex nor concave. The points of optima is $(0, 0, 0)$.

(b) $f_1 = \ln(e^{x_1^2} + e^{x_2^2} + e^{x_3^2})$, $f_2 = e^{x_1} \sin(x_1)\sin(x_2)$

$f_3 = \ln(2 + \sin(x_1) + \cos(x_2))$

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Computing for Gradient of f

$$
\begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\
\frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3}
\end{bmatrix}
$$

$$\frac{\partial f_1}{\partial x_1} = \frac{2x_1 e^{x_1^2}}{e^{x_1^2}+e^{x_2^2}+e^{x_3^2}} \qquad \frac{\partial f_1}{\partial x_2} = \frac{2x_2 e^{x_2^2}}{e^{x_1^2}+e^{x_2^2}+e^{x_3^2}}$$

$$\frac{\partial f_1}{\partial x_3} = \frac{2x_3 e^{x_3^2}}{e^{x_1^2}+e^{x_2^2}+e^{x_3^2}} \qquad \frac{\partial f_2}{\partial x_1} = \sin(x_2)\left[e^{x_1}\cos(x_1) + e^{x_1}\sin(x_1)\right]$$

$$\frac{\partial f_2}{\partial x_2} = e^{x_1}\cos(x_2)\sin(x_1) \qquad \frac{\partial f_2}{\partial x_2} = 0$$

$$\frac{\partial f_3}{\partial x_1} = \left[\frac{\cos(x_1)}{2+\sin(x_1)+\cos(x_2)}\right] \qquad \frac{\partial f_3}{\partial x_2} = \left[\frac{-\sin(x_2)}{2+\sin(x_1)+\cos(x_2)}\right]$$

$$\frac{\partial f_3}{\partial x_3} = 0$$

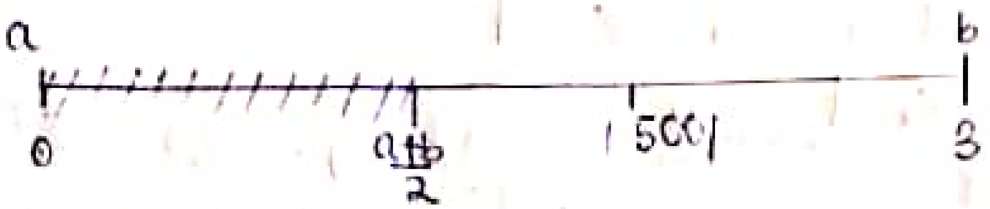
$$
\nabla f = \begin{bmatrix}
\frac{2x_1 e^{x_1^2}}{e^{x_1^2}+e^{x_2^2}+e^{x_3^2}} & \frac{2x_2 e^{x_2^2}}{e^{x_1^2}+e^{x_2^2}+e^{x_3^2}} & \frac{2x_3 e^{x_3^2}}{e^{x_1^2}+e^{x_2^2}+e^{x_3^2}} \\
\sin(x_2)\left[e^{x_1}\cos(x_1)+e^{x_1}\sin(x_1)\right] & e^{x_1}\cos(x_2)\sin(x_1) & 0 \\
\frac{\cos(x_1)}{2+\sin(x_1)+\cos(x_2)} & \frac{-\sin(x_2)}{2+\sin(x_1)+\cos(x_2)} & 0
\end{bmatrix}
$$

Question 2

Given that $f(\lambda) = \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left(1 - \frac{0.5}{1+\lambda^2}\right) + \lambda$,

$0 \le \lambda \le 3$, $\delta = 0.0001$

Stop when interval of Uncertainty is less than 5% of the original.



$\frac{a+b}{2} = \frac{0+3}{2} = 1.5$

$\left(\frac{a+b}{2}\right) - \frac{\delta}{2} = 1.5 - \frac{0.0001}{2} = 1.5000$

$\left(\frac{a+b}{2}\right) + \frac{\delta}{2} = 1.5 + \frac{0.0001}{2} = 1.5001$

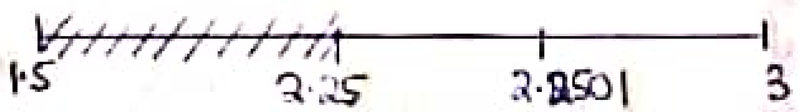
$f(1.5) = 0.2519$

$f(1.5001) = 0.2519$

If $f(1.5) \nless f(1.5001)$, we discard $[0, 1.5]$.

The new interval is $[1.5, 3]$

So, $\frac{\text{New interval}}{\text{Original interval}} \times 100\% = \left(\frac{3-1.5}{3-0}\right) \times 100\% = 50\% \nless 5\%$



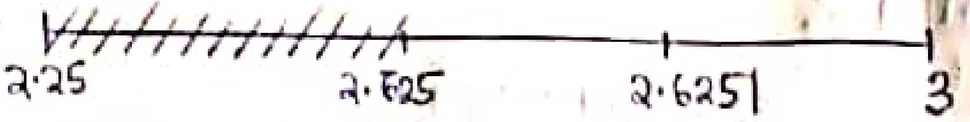
$f(2.25) = 0.1939$

$f(2.2501) = 0.1939$

If $f(2.25) \nless f(2.2501)$, we discard $[1.5, 2.25]$

The new interval is $[2.25, 3]$

So, $\frac{\text{New Interval}}{\text{Original Interval}} \times 100\% = \left(\frac{3-2.25}{3-0}\right) \times 100\% = 25\% \nless 5\%$



$f(2.625) = 0.1720$

$f(2.6251) = 0.1720$

If $f(2.625) \not< f(2.6251)$, we discard $[2.25, 2.625]$

The new interval is $[2.625, 3]$.

So, $\frac{\text{New interval}}{\text{Original}} \times 100\% = \left(\frac{3 - 2.625}{3 - 0}\right) \times 100\% = 12.5 \not< 5\%$

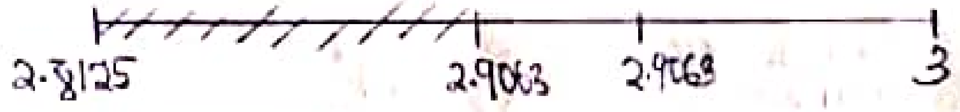
$f(2.8125) = 0.1625$

$f(2.8126) = 0.1625$

If $f(2.8125) \not< f(2.8126)$, we discard $[2.625, 2.8125]$

The new interval is $[2.8125, 3]$

So, $\frac{\text{New interval}}{\text{Original}} \times 100\% = \left(\frac{3 - 2.8125}{3 - 0}\right) \times 100\% = 6.25\% \not< 5\%$



$f(2.9063) = 0.1581$

$f(2.9063) = 0.1581$

If $f(2.9063) \not< f(2.9063)$ we discard $[2.8125, 2.9063]$

The new interval is $[2.9063, 3]$

So, $\left(\frac{\text{New interval}}{\text{original}}\right) \times 100\% = \left(\frac{3 - 2.9063}{3 - 0}\right) \times 100\% = 3.1233\% < 5\%$

Therefore Since $f(3) < f(2.9063)$

$\therefore \bar{x}^* = 3$ and $f(\bar{x}^*) = 0.1540$.

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Question 3

Given $f(x) = 0.65 - \frac{0.75}{1+x^2} - 0.65x\tan^{-1}\left(\frac{1}{x}\right)$, $\Delta x = 0.1$

and $x^{(0)} = 0$

$f[x^{(0)}] = -0.1000$

$x^{(1)} = x^{(0)} + \Delta x = 0 + 0.1 = 0.1$

$f(x^{(1)}) = -0.1882$

If $f(x^{(1)}) < f(x^{(0)})$,

$x^{(2)} = x^{(1)} + 2\Delta x = 0.1 + 2(0.1) = 0.3$

$f(x^{(2)}) = -0.2875$

If $f(x^{(2)}) < f(x^{(1)})$,

$x^{(3)} = x^{(2)} + 2\Delta x = 0.3 + 2(2 \times 0.1) = 0.7$

$f(x^{(3)}) = -0.2902$

If $f(x^{(3)}) < f(x^{(2)})$

$x^{(4)} = x^{(3)} + 2\Delta x = 0.7 + 2(4 \times 0.1) = 1.5$

$f(x^{(4)}) = -0.1541$

If $f(x^{(4)}) \not< f(x^{(3)})$, we relabel the points as

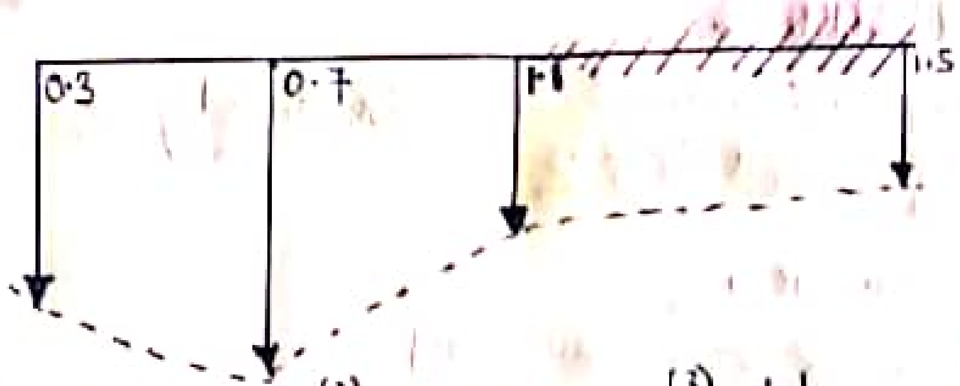
$x^{m} = 1.5$

$x^{m-1} = 0.7$

$x^{m-2} = 0.3$

$\Rightarrow x^{m+1} = x^{m} - \frac{1}{2}\Delta x = 1.5 - \frac{1}{2}(8 \times 0.1) = 1.1$

$f(x^{m+1}) = -0.2169$



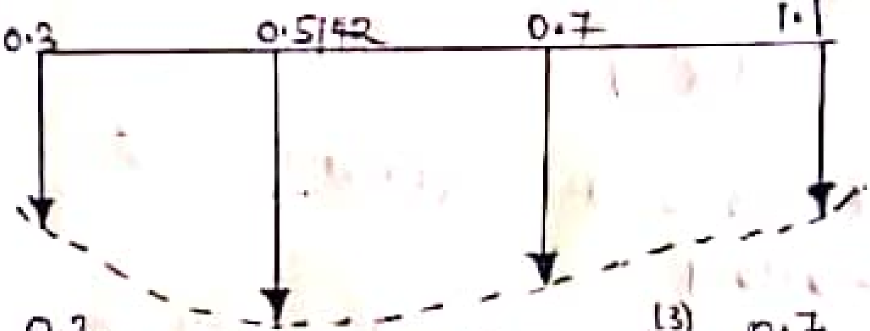
$x^{(1)} = 0.3, \quad x^{(2)} = 0.7, \quad x^{(3)} = 1.1$

$$\Rightarrow \bar{x}_1^* = \frac{1}{2}\left\{\frac{(x^{(1)2} - x^{(2)2})f(x^{(3)}) + (x^{(2)2} - x^{(3)2})f(x^{(1)}) + (x^{(3)2} - x^{(1)2})f(x^{(2)})}{(x^{(1)} - x^{(2)})f(x^{(3)}) + (x^{(2)} - x^{(3)})f(x^{(1)}) + (x^{(3)} - x^{(1)})f(x^{(2)})}\right\}$$

$$= \frac{1}{2}\left\{\frac{(0.3^2 - 0.7^2)(-0.2169) + (0.7^2 - 1.1^2)(-0.2875) + (1.1^2 - 0.3^2)(-0.2902)}{(0.3 - 0.7)(-0.2169) + (0.7 - 1.1)(-0.2875) + (1.1 - 0.3)(-0.2902)}\right\}$$

$\therefore \bar{x}_1^* = 0.5142$

$f(\bar{x}_1^*) = -0.3094$



$x^{(1)} = 0.3, \quad x^{(2)} = 0.5142 \quad x^{(3)} = 0.7$

$$\bar{x}_2^* = \frac{1}{2}\left\{\frac{(0.3^2 - 0.5142^2)(-0.2902) + (0.5142^2 - 0.7^2)(-0.2875) + (0.7^2 - 0.3^2)(-0.3094)}{(0.3^2 - 0.5142)(-0.2902) + (0.5142 - 0.7)(-0.2875) + (0.7^2 - 0.3)(-0.3094)}\right\}$$

$\therefore \bar{x}_2^* = 0.5066$ and $f(\bar{x}_2^*) = -0.3097$

$$|\bar{x}_{k+1}^* - \bar{x}_k^*| = |\bar{x}_2^* - \bar{x}_1^*| = |0.5066 - 0.5142| = 0.0076 < 0.01$$

Therefore, $\bar{x}^* = 0.5066$ and $f(\bar{x}^*) = -0.3097$

Section B is also worth 50 marks. It has 3 numerical problems which require more extensive calculations. In order to obtain full credit for any question, all work must be shown in detail. Full credit will not be given for an answer not supported by working.

SECTION B

1. (a) Define convex function? Hence examine whether the following function is convex or concave and find the points of optima:

$$f(x_1,x_2,x_3)=x_1^2+2x_2^2-7x_3^2-4x_1x_2+6x_1x_3-5x_2x_3$$

(b) Compute the Gradient and Hessian if $f_1=\ln\left(e^{x_1^2}+e^{x_2^2}+e^{x_3^2}\right)$, $f_2=e^{x_1}\sin(x_1)\sin(x_2)$ and $f_3=\ln(2+\sin(x_1)+\cos(x_2))$

2. Find the minimum of the function $f(\lambda)=\frac{0.5}{\sqrt{1+\lambda^2}}-\sqrt{1+\lambda^2}\left(1-\frac{0.5}{1+\lambda^2}\right)+\lambda$ using the dichotomous search method in the interval $[0,3]$ using a value of $\delta=0.0001$. Stop when the interval of uncertainty is less than 5% of the original.

3. Minimize $f(x)=0.65-\frac{0.75}{1+x^2}-0.65x\tan^{-1}\frac{1}{x}$ with a step size of 0.1 starting from point 0.0 using the Coggin's algorithm. Stop when $|\overline{x}_{k+1}-\overline{x}_k|<0.01$.

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