UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

SECOND SEMESTER EXAMINATIONS, MAY. 2016

COURSE NO : MA 376
COURSE NAME : OPTIMIZATION TECHNIQUES
CLASS : MA **III** TIME: **3 HOURS**

Name: ____ Index Number: _______________

INSTRUCTIONS: During this exams, any communication with any person (other than the instructor) in any form, including written, signed, verbal, or digital, is understood to be a violation of academic integrity.

The scoring for this exam is determined by the formula $\left[C-(0.25\times I)\right]\times 1.4$ where C is the number of correct responses and I is the number of incorrect responses. An unanswered question earns zero points. IDENTIFY and CIRCLE the best possible answer on the question booklet.

1. Which of the following statements is incorrect regarding the Equal Interval Search and Golden Section Search methods?
   A. Both methods require an initial boundary region to start the search
   B. The number of iterations in both methods are affected by the size of ε
   C. Everything else being equal, the Golden Section Search method should find an optimal solution faster.
   D. Everything else being equal, the Equal Interval Search method should find an optimal solution faster.
   E. None of the above

Use this information to answer questions **2** to **5**.
Given $f(x)=x^4-4x+1,\ \Delta x=0.5$ and $x^{(0)}=0.0,$ can be minimized using the D.S.C algorithm.

2. $x^{(1)}=$ **A.** 0 **B.** 0.5 **C.** 1 **D.** 1.5 **E.** 2
3. $f\left(x^{(1)}\right)=$ **A.** 0.9375 **B.** -0.9375 **C.** 1.9375 **D.** -1.9375 **E.** 2.9375
4. $f\left(x^{(2)}\right)=$ **A.** -0.44 **B.** -0.94 **C.** -2 **D.** 0.0625 **E.** 0.625
5. $\tilde{x}^*=$ **A.** 1.08 **B.** 1.00 **C.** 1.05 **D.** 0.05 **E.** 0.06

6. Which of the following parameters is not required to use the Golden Section Search method for optimization?
   A. The lower bound for the search region **B.** The upper bound for the search region
   C. The golden ratio **D.** The function to be optimized
   E. None of the above

Use this information to answer Questions **7** to **11**.
$f(x)=x^4-4x+1$ subject to $0\le x\le 2$ can be minimized using the cubic search algorithm, where

$f(x)=g(x)=a+b(x-x_1)+c(x-x_1)(x-x_2)+d(x-x_1)^2(x-x_2)$.

7. Find the value of a **A.** 0 **B.** 1 **C.** -1 **D.** 2 **E.** -2
8. Find the value of b **A.** 1 **B.** 2 **C.** 3 **D.** 4 **E.** 5
9. Find the value of c **A.** 1 **B.** 2 **C.** 3 **D.** 4 **E.** -3
10. Find the value of d **A.** 4 **B.** 3 **C.** 2 **D.** 1 **E.** 0
11. Find the value of $\tilde{x}^*$ **A.** -0.33 **B.** 0.33 **C.** 1 **D.** -1 **E.** 0
12. The Fibonacci series, is given by the expression

A. $\dfrac{\left(\sqrt{5}+1\right)^{k+1}+(-1)^k\left(\sqrt{5}-1\right)^{k+1}}{\sqrt{5}2^{k+1}}$ **B.** $\dfrac{\left(\sqrt{5}+1\right)^{k+1}+(-1)^k\left(\sqrt{5}-1\right)^{k+1}}{2^{k+1}}$

C. $\dfrac{\left(\sqrt{5}+1\right)^{k+1}+(-1)^k\left(\sqrt{5}-1\right)^{k+1}}{\sqrt{5}2^{k}}$ **D.** $\dfrac{\left(\sqrt{5}+1\right)^{k}+(-1)^k\left(\sqrt{5}-1\right)^{k}}{\sqrt{5}2^{k+1}}$

E. $\dfrac{\left(\sqrt{5}+1\right)^{k}+(-1)^{k+1}\left(\sqrt{5}-1\right)^{k}}{\sqrt{5}2^{k}}$

13. If $f_1=x^2y+2x+z-2=0$, $f_2=\cos x+z\sin y+x-1=0$, and $f_3=e^{xy}+y\sin z=0$, then $\dfrac{\partial(f_1,f_2,f_3)}{\partial(x,y,z)}=$

A. $\begin{bmatrix} 2xy & x^2 & 1 \\ -\sin x & z\cos y & \sin y \\ ye^{xy} & xe^{xy} & y\cos z \end{bmatrix}$ **B.** $\begin{bmatrix} 2xy+2 & x^2 & 1 \\ -\sin x+1 & z\cos y & \sin y \\ ye^{xy} & xe^{xy}+\sin z & y\cos z \end{bmatrix}$ **C.** $\begin{bmatrix} 2xy-2 & x^2 & 1 \\ -\sin x+1 & z\cos y & \sin y \\ ye^{xy} & xe^{xy} & y\cos z \end{bmatrix}$

D. $\begin{bmatrix} xy+1 & 2x & -1 \\ -\sin x & z\cos y & \sin y \\ ye^{xy} & xe^{xy} & y\cos z \end{bmatrix}$ **E.** $\begin{bmatrix} 2xy+2 & x^2 & 1 \\ -\sin x+1 & z\cos y & \sin y \\ ye^{xy} & xe^{xy}+\sin z & y\cos z \end{bmatrix}$

14. At the k^{th} stage of the Davies, Swann and Campey's (D.S.C) algorithm with $x^{(k)}$ known, we compute $x^{(k+1)}$ as

A. $x^{(k)}+2\Delta x$ **B.** $x^{(k)}+\Delta x$ **C.** $x^{(k)}-2\Delta x$ **D.** $x^{(k)}-\Delta x$ **E.** $x^{(k)}+\frac{1}{2}\Delta x$

15. Using the D.S. C, if $f\left(x^{(1)}\right)>f\left(x^{(2)}\right)$, where Δx is the initial step length, we set $x^{(1)}$ to be equal to

A. $x^{(0)}-\Delta x$ **B.** $x^{(0)}-2\Delta x$ **C.** $x^{(1)}-\Delta x$ **D.** $x^{(0)}+\Delta x$ **E.** None

16. When applying the Golden Section Search method to a function $f(x)$ to find its maximum, the $f(x_1)>f(x_2)$ condition holds true for the intermediate points x_1 and x_2.Which of the following statements is incorrect?

A. The new search region is determined by $[x_2,x_u]$

B. The Intermediate point x_1 stays as one of the intermediate points

C. The upper bound x_u stays the same

D. The new search region is determined by $[x_l,x_1]$

17. In Dichotomous search we evaluate two functional values in the interval $[a,b]$ where ε is a positive number at

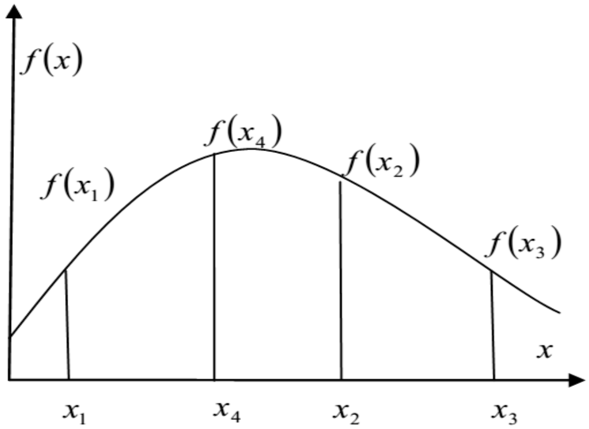
A. $\frac{b-a}{2}-\frac{\varepsilon}{2}$ and $\frac{b-a}{2}+\frac{\varepsilon}{2}$ **B.** $\frac{b-a}{2}-\varepsilon$ and $\frac{b-a}{2}+\varepsilon$ **C.** $\frac{b-a}{2}-\frac{1}{2}$and $\frac{b-a}{2}+\frac{1}{2}$

D. $\frac{b-a}{2}-\frac{\varepsilon}{2}$ and $\frac{b-a}{2}-\frac{3\varepsilon}{2}$ **E.** $\frac{b+a}{2}-\frac{\varepsilon}{2}$and $\frac{b+a}{2}+\frac{\varepsilon}{2}$

18. The three-equal interval search algorithm divides the interval of uncertainty into … equal parts.

A. 3 **B.** 4 **C.** 5 **D.** 6 **E.** 7

19. In the graph below, the lower and upper boundary of the search is given by x_1 and x_3 respectively. If x_4 and x_2 are the initial intermediary points, which of the following statement is false?



A. The distance between x_2 and x_1 is equal to the distance between x_4 and x_3
B. The distance between x_4 and x_2 is approximately 0.618 times the distance between x_2 and x_1
C. The distance between x_4 and x_1 is approximately 0.618 times the distance between x_4 and x_3
D. The distance between x_4 and x_1 is equal to the distance between x_2 and x_3

Use this information to answer Questions **20** to **23**.
We wish to use the three–point equal interval search method for the minimization of the unimodal function $f(x)=\frac{1}{6}x^6-x+\frac{1}{x^2+1}$ in the interval $-2\le x\le 2$, reducing the interval of uncertainty to less than 10%.

20. How many evaluations are required?
A. 4 **B.** 6 **C.** 8 **D.**10 **E.** 12

21. The first three points where evaluations are carried out are
A. -1, 0, 1 **B.** $\frac{-1}{2},0,\frac{1}{2}$ **C.** $\frac{-1}{3},0,\frac{1}{3}$ **D.** $\frac{-1}{4},0,\frac{1}{4}$ **E.** $\frac{-1}{5},0,\frac{1}{5}$

22. The second set of points where evaluations are carried out are
A. -0.5 and -1.5 **B.** 0.5 and 1.5 **C.** -0.5 and 0.5 **D.** -1.5 and 0.5 **E.** -0.5 and 1.5

23. The third set of points where evaluations are carried out are
A. 1.25 and 1.75 **B.** 0.75 and 1.25 **C.** -1.25 and 0.25 **D.** -0.25 and 0.25 **E.**-0.25 and 1.25

Use the information below to answer Questions **24** and **25**.
Using three-point equal search algorithm,

24. Its efficiency is
A. $\frac{1}{2^{(m-1)/2}}$ **B.** $\frac{1}{2^{(m)/2}}$ **C.** $\frac{1}{2^{m-1}}$ **D.** $\frac{1}{2^{m-2}}$ **E.** $\frac{1}{2^{m}}$

25. The length of the interval of uncertainty after n applications is
A. $\frac{b-a}{3^n}$ **B.** $\frac{b-a}{2^{n-2}}$ **C.** $\frac{b-a}{2^{n-1}}$ **D.** $\frac{b-a}{2^n}$ **E.** $\frac{b+a}{2^{n-1}}$

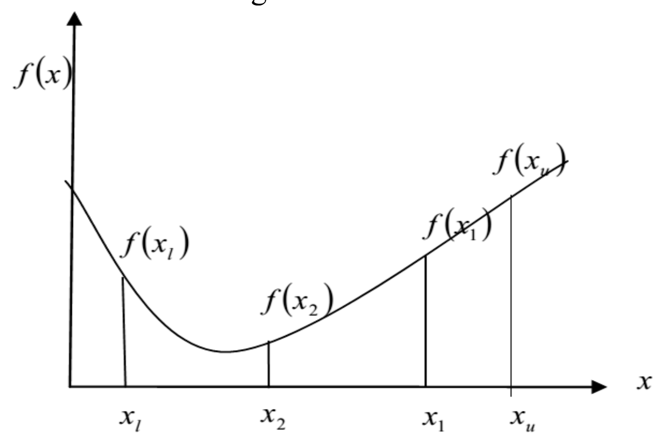
26. Using the Golden Section Search method, find two numbers whose sum is 90 and their product is as large as possible. Conduct two iterations on the interval [0, 90].
A. 30 and 60 **B.** 45 and 45 **C.** 38 and 52 **D.** 20 and 70 **E.** None of the above

27. Which of the following statements is not true?
A. Sequential search methods are applicable only to unimodal functions
B. Dichotomous search is a sequential search method
C. Two-point equal interval search is a sequential search method

D. D.S.C is a simultaneous search method

E. The efficiency of any search method is computed by $\frac{\Delta^{(n)}}{\Delta^{(0)}}$, where $\Delta^{(n)}$ is the interval of uncertainty after the n functional evaluations and $\Delta^{(0)}$ the original interval of uncertainty.

28. Consider the problem of finding the minimum of the function shown below. Given the intermediate points in the drawing, what would be the search region in the next iteration?



A. $[x_2, x_u]$ **B.** $[x_1, x_u]$ **C.** $[x_l, x_1]$ **D.** $[x_l, x_2]$ **E.** $[x_1, x_2]$

Use the information below to answer Questions **29** to **34**.

Using the Fibonacci search for the minimum value of the function $f(x) = \frac{(x^6 - x + 1)}{e^{x^2+1}}$ in the interval $-1 \le x \le 1$, reducing the interval of uncertainty to less than $2\,\%$ (using the usual notations),

29. How many evaluations are required to solve this problem?
A. 24 **B.** 19 **C.** 14 **D.** 9 **E.** 6

30. Evaluate $x_{u,1}$	**A.** -0.236364	**B.** 0.25	**C.** 0.50	**D.** 0.743	**E.** 0.236346
31. Evaluate $f(x_{l,1})$	**A.** 0.265722	**B.** 0.43018	**C.** -0.265722	**D.** -0.43018	**E.** 0.5618
32. Evaluate $f(x_{u,1})$	**A.** 0.265722	**B.** 0.43018	**C.** -0.265722	**D.** 0.5618	**E.** -0.5618
33. Evaluate I_2	**A.** 0. 472728	**B.** 1.472728	**C.** 0.7472	**D.** 0.7878	**E.** 1.23636
34. Evaluate $x_{u,2}$	**A.** 0.617647	**B.** -0.5227	**C.** -0.52727	**D.** 0.52727	**E.** 0.431469

Use this information to answer Questions **35** to **46**.

Random search method can be used to minimize the function $f(x) = x^2 \sin\frac{1}{x} - \frac{1}{x}\cos x$ in the interval $1 \le x \le 10$. 0.48, 0.87, 0.77, 0.96, 0.43, 0.39, 0.76, 0.93, 0.08 and 0.79 are ten random numbers $y \in [0,1]$ that can be used for this purpose.

35. Evaluate $g(0.48)$	**A.** -1.64711	**B.** 1.64711	**C.**-0.94309	**D.**5.32000	**E.** 1.74711
36. Evaluate $g(0.87)$	**A.** 8.83	**B.** -8.83	**C.** -0.050487	**D.** 0.042204	**E.** -8.83
37. Evaluate $g(0.77)$	**A.** 0.36126	**B.**-0.36126	**C.**-7.93	**D.**7.93	**E.** 0.O13507
38. Evaluate $g(0.96)$	**A.** 0.0598	**B.** 9.64	**C.** -9.64	**D.** 0.06598	**E.** 0.198152
39. Evaluate $g(0.43)$	**A.**-1.97919	**B.** -4.87	**C.** 4.87	**D.** -4.80362	**E.** -0.1196

40. Evaluate $g(0.39)$	**A.**-2.28853	**B.** -4.51	**C.** 4.51	**D.**-4.51771	**E.** -0.142329
42. Evaluate $g(0.76)$	**A.** -7.81698	**B.** 0.010475	**C.** 7.84	**D.** -7.84	**E.** -0.39481
43. Evaluate $g(0.93)$	**A.** 9.37	**B.** -9.37	**C.** 0.058238	**D.** -9.45879	**E.** 0.118036
44. Evaluate $g(0.08)$	**A.**-12.4604	**B.**-1.72	**C.** 1.72	**D.** -0.55111	**E.**-17115
45. Evaluate $g(0.79)$	**A.**-8.11	**B.** 8.35669	**C.** 8.11	**D.** 0.019475	**E.** -8.35669
46. Evaluate $\min f$	**A.** -12.4604	**B.** -0.94309	**C.** -1.7115	**D.** -8.35669	**E.** -9.45879

Use the information below to answer Questions **47** to **50**.
Using Powell's algorithm to minimize $f(x) = x^4 - 4x - 1, x \in \Re$, with TOL=$\varepsilon = 0.001$, $x^{(1)} = 0.0$ and $\Delta x = 0.5$

47. Find $f(x^{(2)})$	**A.** -1	**B.** -2.9375	**C.** -4	**D.** -3.078	**E.** -3.35
48. Find $f(x^{(3)})$	**A.** -1	**B.** -2.9375	**C.** -4	**D.** -3.078	**E.** -3.35
49. Find $\tilde{x}^*$	**A.** -1.25	**B.** -1.05	**C.** -1.35	**D.** 1.35	**E.** 0.5
50. Find $f(\tilde{x}^*)$	**A.** 6.44	**B.** 4.42	**C.** 7.72	**D.** -3.08	**E.** -2.94

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