**UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA**

END OF FIRST SEMESTER EXAMINATIONS DECEMBER, 2018



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| COURSE NO. | : CE/EL/GL/GM/MC/MN/MR/PE/ES 459 |
| COURSE NAME | : Operations Research |
| CLASS | : CE IV, EL IV, GL IV, GM IV, MC IV, MN IV, MR IV, PE IV, ES IV |
| TIME | : 3 Hours |

**ANSWER ANY FOUR (4) QUESTIONS**

**Q 1**

A machine manufacturing company produces magnetic machines for industrial applications. Machines are produced from two production plants (Plant 1, Plant 2). Both production plants produce two types of machine: light duty and heavy duty machine. The output from each production plant and the time available for production on each plant are given in Table 1.

**Table 1**

| | Number of light duty machine produced per hour | Number of heavy duty machine produced per hour | Time available (hr/week) |
|---|---|---|---|
| Plant 1 | 10 | 6 | 200 |
| Plant 2 | 16 | 10 | 240 |

The company uses steel and aluminium as the primary raw materials for producing the machines. The quantity of materials needed to produce one unit of machine on each plant is given in Table 2.

**Table 2**

| | **Light duty machine** | | **Heavy duty machine** | |
|---|---|---|---|---|
| | Steel (kg) | Aluminium (kg) | Steel (kg) | Aluminium (kg) |
| **Plant 1** | 15 | 8 | 10 | 15 |
| **Plant 2** | 9 | 10 | 20 | 19 |

The company currently has 35,000 kg of steel and 30,000 kg of aluminium available for each week. A profit of $100 and $150 is made on each unit of light and heavy duty magnetic machine respectively. Sales forecasts indicate that a maximum of 1500 light duty machine and a minimum of 250 heavy duty machine can be sold weekly. The company has a policy of dedicating at least 50% of monthly production time to the production of heavy duty machines which yield higher profits. It also wants Plant 2 to produce at least 60% of all machines produced in a week.

Management of the company wants to know the number of hours that should be allocated for production of machine on each plant to maximise profits. Formulate this as a Linear Programming model (in standard form). [20]

What is a model? [1.5]
What is proportionality assumption in linear programming? [2]
What is supply in the transportation problem? [1.5]

**Q 2**

a. A linear programming model below represents the production process of product 1 and product 2 to maximise profit from the use three resources: labour, steel and copper. The decision variables $X_1$ and $X_2$ are defined as the number of units of products 1 and 2 produced respectively. Other details are indicated in the linear programming model.

| | |
|---|---|
| Max $Z = 90X_1 + 150X_2$ | Profit ($) |
| Such that: | |
| $4X_1 + 6X_2 \leq 30$ | labour (hr) |
| $8X_1 + 2X_2 \leq 40$ | Steel (kg) |
| $6X_1 + 2X_2 \geq 24$ | Copper (kg) |
| $X_1, X_2 \geq 0$ | |

i. Solve the above linear programming model using the simplex method [18]
ii. Interpret the optimal solution [3]
iii. Define the variables for the dual form of this model [2]
iv. What are the shadow prices? [2]

**Q 3a**

The Simplex method was used to solve the following linear programming model with the optimal tableau shown in Table 3:

Max $Z = 0.06X_1 + 0.08X_2$

Subject to:

$X_1 + X_2 \leq 400$

$X_2 \leq 250$

$5X_1 + 10X_2 \leq 3000$

$X_1, X_2 \geq 0$

**Table 3**

| $C_j$ / $C_b$ | basis | $b_i$ | 0.06 $X_1$ | 0.08 $X_2$ | 0 $S_1$ | 0 $S_2$ | 0 $S_3$ |
|---|---|---|---|---|---|---|---|
| 0 | $S_2$ | 50 | 0 | 0 | 1 | 1 | -1/5 |
| 0.08 | $X_2$ | 200 | 0 | 1 | -1 | 0 | 1/5 |
| 0.06 | $X_1$ | 200 | 1 | 0 | 2 | 0 | -1/5 |
| | $Z_j$ | 28 | 0.06 | 0.08 | 0.04 | 0 | 0.004 |
| | $C_j$-$Z_j$ | | 0 | 0 | -0.04 | 0 | -0.004 |

Use sensitivity analysis to find the range of values of the profit coefficient of $X_1$ for which the current optimal basis remains optimal. [8]

**Q 3b.**

Ghacem is a cement manufacturing company in Ghana with two manufacturing plants located in Accra and Kumasi from which it supplies cement to three mine site (Mine 1, Mine 2 and Mine 3). The yearly production capacities of the plants located in Accra and Kumasi are 700 t and 900 t respectively. The yearly cement requirements (demands) of Mine 1, Mine 2 and Mine 3 are 350 t, 650 t and 450 t respectively.

The cost ($) of transporting a tonne of cement from each manufacturing plant to a construction company is given in Table 2:

**Table 4**

| Plant | Yearly Supply (t) | Mine | Yearly Requirement (t) |
|---|---|---|---|
| **Accra** | 700 | Mine 1 | 400 |
| **Kumasi** | 900 | Mine 2 | 650 |
| | | Mine 3 | 450 |

**Table 5**

| To Mine / From Plant | Mines | | |
|---|---|---|---|
| | **Mine 1** | **Mine 2** | **Mine 3** |
| **Accra** | 30 | 25 | 29 |
| **Kumasi** | 25 | 33 | 28 |

i. Use the Northwest Corner Method or Vogel's Approximation method to determine the initial distribution of lime from the plants to the mines.

ii. Determine the distribution that will minimise the total yearly cost of transportation. State the minimum total yearly cost of transportation.

[17]

**Q 4a**

Formulate the dual of the following linear programming model: [8]

Max $Z = 25X_1 + 15X_2 - 32X_3$

Such that:

$32X_1 - 18X_2 + 19X_3 \geq 35$

$25X_2 + 30X_3 = 40$

$14X_1 + 19X_2 - 16X_3 \leq 36$

$X_1, X_2 \geq 0$

$X_3$ – unrestricted

**Q 4b.**

A project for the installation of a large mining excavator involves eleven (11) activities. The three activity time estimates and the precedence relationships are shown in Table 6.

**Table 6**

| Activity | Activity Predecessor | Estimated Activity Times (days) | | |
|---|---|---|---|---|
| | | Optimistic | Most Likely | Pessimistic |
| A | - | 3 | 5 | 7 |
| B | A | 5 | 7 | 9 |
| C | A | 4 | 6 | 8 |
| D | C | 3 | 4 | 5 |
| E | B, D | 5 | 6 | 7 |
| F | C | 2 | 3 | 4 |
| G | B | 6 | 8 | 10 |
| H | F | 5 | 7 | 9 |
| I | E, G, H | 4 | 6 | 8 |
| J | B | 4 | 7 | 10 |

| | | | | |
|---|---|---|---|---|
| K | F | 1.5 | 2 | 2.5 |

i. Construct a PERT network representing the installation project. [9]
ii. Determine the critical activities and the minimum duration of the project. [3]
iii. Determine the total slack of all non-critical activities. [2]
iv. What is the probability of completing the project in 25 days? [3]

**Q 5a**

Four (4) drilling machines are used for drilling in a mining operation. Each of five (5) operators can operate each of the drilling machines. The performance of the operators is measured by the number of minutes a normal schedule of drilling operation is completed when a given machine is operated. Table 7 shows the number of minutes used if each operator is assigned to a machine.

**Table 7**

| | **MACHINE** | | | | |
|---|---|---|---|---|---|
| **OPERATOR** | **1** | **2** | **3** | **4** | **5** |
| **A** | 15 | 17 | 19 | 16 | 14 |
| **B** | 20 | 15 | 18 | 15 | 19 |
| **C** | 18 | 17 | 17 | 19 | 16 |
| **D** | 20 | 18 | 18 | 21 | 16 |

i) Use the assignment method to find the optimal assignment of supervisors to crews
ii) Determine the total repair time involved in the optimal assignment
iii) Determine at least one additional optimal assignment, if any.

[10]

**Q5 b.**

Workers arrive at a security check point in a manner described by a discrete random variable x (Table 8). The workers slot their ID card through a security check machine and are allowed passage. Since the check is machine operated, the time spent is constant at 30 seconds per worker. There is only one security check point so workers wait for their turn when another worker is in the process of security checks. The queue discipline is first come first served.

**Table 8**

| Time between Arrivals (seconds) (x) | Frequency of Occurrence |
|---|---|
| 10 | 70 |
| 20 | 60 |
| 35 | 45 |
| 50 | 10 |
| 55 | 15 |

i. Develop a random process generator for time between arrivals. [4]
ii. With a seed value of 37, a constant multiplier of 17 and modulo of 83, use the Multiplicative Congruential technique to generate 8 random numbers. [2]
iii. Simulate the next eight arrivals based on the random numbers and process generator. You may use the table (Table 9) headings provided as a guide. [6]

Table 9

| $i^{th}$ Vehicle | Random number | Time between arrivals (s) x | Arrival clock | Wait time (s) | Queue length at entry | Check time (s) | Time to Depart the check point (s) | Time at check point (s) |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |

iv. Find the mean waiting time, maximum queue length and mean time spent at the security check point. [3]

**Q 6a**

Solve the following Integer Programming (IP) model using the branch and bound method (use the graph sheet provided):

Min $Z = 40X_1 + 24X_2$

Such that:

$4X_1 + 2X_2 \geq 52$

$6X_1 + 8X_2 \geq 80$

$X_1, X_2 \geq 0$ and integer

[20]

**Q 6b**

i. What is meant by multiple optimal solutions in a pure integer programming problem? [1.5]

ii. Explain why for a minimisation problem the optimal objective function value to a given integer programming model is always more than or equal to the optimal objective function value of the corresponding relaxed linear programming model? [2]

iii. What is a convex set of points with respect to linear programming? [1.5]

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