UNIVERSITY OF MINES AND TECHNOLOGY – TARKWA

SECOND SEMESTER EXAMINATION, MAY/JUNE, 2018

COURSE NO: MA 474

COURSE NAME: **TIME SERIES AND FORECASTING II**

CLASS: MA IV TIME: 3 HOURS

ANSWER QUESTION 1 AND ANY OTHER TWO QUESTIONS.
ALL QUESTIONS CARRY EQUAL MARKS

Q1. (a). Explain the term white noise in the time series process.

(b). Suppose $Y_t = \frac{e_t + e_{t-i}}{2}$ where e's are assumed to be independently and identically distributed with zero mean and variance. Show that;

i. $Var(Y_t) = 0.5\sigma_e^{\,2}$

ii. Autocovariance function $\gamma_{t,\ t-1} = 0.25\sigma_e^{\,2}$ for all *t*.

(c). Let $\{Y_t\}$ be a sequence of independent normal random variables, each with mean 0 and variance σ^2. Which, if any, of the following processes is stationary? Explain your answer and state its partial autocorrelation function (PACF) coefficient at lag two.

i. $Y_2 = 0.64 + 1.34Y_{t-1} + 2.06Y_{t-2} + \varepsilon_t$

ii. $Y_2 = 0.64 + 0.85Y_{t-1} + 0.04Y_{t-2} + \varepsilon_t$

(d). The following table presents a time series data on an unemployment rate in ASA Community for the year 2000-2008. Fill in the values under the following headings.

Year	Unemployment rate	Lag one	Lead one	Difference one	Difference three
2000	12				
2001	10				
2002	15				
2003	8				
2004	11				
2005	20				
2006	13				
2007	7				
2008	18				

Q2. (a). State one major difference between the lead of a time series variable and the lag of time series variable.

(b). Consider the second order Autoregressive model AR (2) $Y_t = \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + e_t$ where e_t is independent of $Y_{t-1}, Y_{t-2},\ldots\ldots$ .Assuming that the process is stationary with zero mean and constant variance. If the Autocovariance function generated is $\gamma_k = \theta_1 \gamma_{k-1} + \theta_2 \gamma_{k-2}$ for k = 1,2.3……..

i. Establish the Yule Walker equation $\rho_k = \theta_1 \rho_{k-1} + \theta_2 \rho_{k-2}$ for k = 1,2,3..

ii. Hence or otherwise, prove that $\rho_2 = \dfrac{\theta_2 (1-\theta_2) + \theta_1^2}{1-\theta_2}$ for k = 2.

(c). Let $\{Y_t\}$ be the moving average process of order two given by

$$Y_t = \varepsilon_t - \phi_1 \varepsilon_{t-1} - \phi_2\ \varepsilon_{t-2}$$

Where $\{Y_t\}$ is WN (0, 1). Find the autocorrelation functions at lag one and lag two when $\phi_1 = 0.4$ and $\phi_2 = 0.7$

Q3. (a).Explain the difference between the serial correlation and white noise of a time series process.

(b). Let $\{Y_t\}$ be AR (2) process defined. Assuming that the process is stationary and that the variance at lag k is given by $\gamma_k = \phi_1 \gamma_{k-1} + \phi_2 \gamma_{k-2}$. Show that

i. $\rho_1 = \dfrac{\phi_1}{1-\phi_2}$

(c). If $\{Y_t\}$ is random walk defined by $Y_t = \sum_{i=1}^{n} \varepsilon_t$ such that $Y_1 = \varepsilon_1$, and $Y_2 = Y_1 + \varepsilon_2 = \varepsilon_1 + \varepsilon_2$. Show that $E(Y_t^2) = t\sigma^2 < \infty$ for all t, and for $k \geq 0$

Q4. (a) Briefly explain the following terms in the time series

i. Random Walk

ii. Purely Random Process

iii. Stochastic Process

(b). Consider the model $X_t = \varepsilon_t - \theta\varepsilon_{t-1}$ where X_t is the Moving Average process of order one MA (1). Show that the following statements are valid.

$$\left.\begin{aligned} & E(X\ ) = 0 \\ & \gamma_0 = Var(X_t) = \sigma_\varepsilon^{2}(1+\theta^2) \\ & \gamma_1 = -\theta\sigma_\varepsilon^{2} \\ & \rho_1 = (-\theta)/(1+\theta^2) \end{aligned}\right\}$$

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