Statistical Inferences, MA 380

<u>Answer three (3) Questions Only</u> **Time: 3 Hours**

1) Consider the three (3) sample observations, $(X_1 = 0.4, X_2 = 0.7, X_3 = 0.9)$ obtained from a continuous distribution with density

$$f(x) = \theta x^{\theta-1} \quad \text{for } 0 < x < 1$$

   a) What is the basic principle of the method of moments?

   b) Estimate θ by the method of moments.

2) Let $X \sim N(\mu, \sigma^2)$, where $-\infty < \mu < \infty$ and σ^2 is known with a given probability density function

$$f(X \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

   a) Estimate the parameters μ and σ^2, using the maximum likelihood method.

   b) Demonstrate whether $\hat{\sigma}^2$ is a bias estimator of σ^2 or not.

3) Suppose Y is a scalar observation drawn from a parameterised Poisson (λ) distribution

$$P_Y(y;\lambda) = \frac{\lambda^{y_i} e^{-\lambda}}{y_i!}$$

for $y = 0, 1, 2, \ldots$

   a) Find the Fisher information $I(\lambda)$ and the Cramer-Rao Lower Bound (CRLB) for estimating the scalar parameter λ.

   b) Find the Minimum Variance Unbiased estimator that achieves the CRLB in this case.

4) Consider the sample $2.5, 6.8,$ and 3.0 from an exponential distribution

$$f(x \mid \theta) = \frac{1}{\theta} \exp\left(-\frac{x}{\theta}\right); \quad x > 0$$

(a) (i) Use the Likelihood Ratio Test to perform the following hypothesis test:

$$H_0 : \theta = \theta_0 \text{ versus } H_1 : \theta = \theta_1 \text{, where } \theta_1 < \theta_0$$

at $\alpha = 5\%$ significance level.

(ii) Suppose $H_0 : \theta = 2$ and $H_1 : \theta = 1$, by using the sample $2.5, 6.8,$ and 3.0, demonstrate whether the null hypothesis will be accepted or rejected at $\alpha = 0.05\%$ significance level .

5) Let $X_1, X_2, ..., X_n$ be a random sample of size n from a normal population with mean μ and variance σ^2. Consider

$$\hat{\mu}_1 = \frac{1}{2}\left(\mathrm{X}_1 + \mathrm{X}_2\right)$$

$$\hat{\mu}_2 = \frac{1}{4}\mathrm{X}_1 + \frac{\left(\mathrm{X}_2 + ... + \mathrm{X}_{n-1}\right)}{2(n-2)} + \frac{1}{4}\mathrm{X}_n$$

$$\hat{\mu}_3 = \overline{\mathrm{X}} .$$

a) Show whether the maximum likelihood estimator $\hat{\sigma}^2$ is an unbiased estimator of the parameter σ^2 or not.

b) Find $eff\left(\hat{\mu}_3, \hat{\mu}_2\right)$ and $eff\left(\hat{\mu}_3, \hat{\mu}_1\right)$.

Examiner

(Paul Boye)