



UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV. / DEC. 2018

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COURSE NO: MA 379

COURSE NAME: ELEMENTS OF TOPOLOGY

CLASS: MA III

TIME: 3 HOURS

Name: _____ Index Number: _____

Answer all questions

1. What is the difference between a closed ball and an open ball? [2 marks]
2. Define a closed set in terms of a sequence of a point. [2 marks]
3. Define degenerate interval and give one example. [3 marks]
4. Define the boundary point of a set? [2 marks]
5. Why is the open set the largest set contained in a set? [2 marks]
6. What is the difference between a metric and a norm defined on a space? [2 marks]
7. Give an example of a metric defined on a continuous space and prove it. [9 marks]
8. Why is the open ball the same as the open set in metric spaces? [2 marks]
9. Define the standard topology. [2 marks]
10. Define the discrete metric. [2 marks]
11. State and prove the Holder's inequality. [10 marks]
12. Given that $A = [-1, 1]$ is the standard topology on R
find (i) $cl(A)$ (ii) $int(A)$ (ii) ∂A [3 marks]

13. Given that X is a metric space prove that every open ball is an open set. [7 marks]
14. Let $f: X \rightarrow Y$ be a function from a metric space (X, d) to another metric space (Y, ρ) and let $x_0 \in X$ then prove that f is continuous at x_0 if and only if for every sequence $\{x_n\}$ $x_n \rightarrow x_0$, and $f(x_n) \rightarrow f(x_0)$. [10 marks]
15. Let (X, d) and (Y, ρ) be metric spaces. A function $f: X \rightarrow Y$ is continuous if and only if $f^{-1}(F)$ is closed in X whenever F is closed in Y . [10 marks]
16. State and prove the Bolzano-Weierstrass theorem. [10 marks]
17. Define Connectedness [2 marks]
18. State five criteria for connectedness [10 marks]
19. Define Homeomorphism [2 marks]
20. Let $X = (-1, 1)$ and $Y = \mathbb{R}$, given that a continuous map $f: (-1, 1) \rightarrow \mathbb{R}$ is defined by $f(x) = \sin\left(\frac{\pi x}{2}\right)$ then determine whether X is homeomorphic to Y . [8 marks]

Examiner: Henry Otoo