UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV/DEC 2018

COURSE NO: MA 377

COURSE NAME: NUMERICAL METHODS FOR ODE

CLASS: MA III **TIME:** 3 HOURS

Name: ____ Index Number: _______________

SECTION ONE

INDICATE either TRUE or FALSE against each question in this section.

1. Modified Euler's and Heun's methods are all second-order Runge-Kutta method?
2. Euler's method is an improvement on the point-slope method?
3. Heun's method can be considered a simple form of the predictor – correction method?
4. Single step methods are not self-starting.
5. Multistep methods are not self-starting.
6. Euler's method is the most efficient compared with most other numerical methods.
7. The accuracy of second-order Runge-Kutta methods is less than the accuracy of Euler's method.
8. Adams-Bashforth methods require either the solution of a nonlinear equation or a predictor-corrector scheme
9. The second-order Adams-Moulton method is the trapezoid method
10. The Runge-Kutta methods can be used to generate starting values for Adams methods.

SECTION TWO

CIRCLE the best possible answer on the question booklet in this section.

11. Taylor series will be very useful to give some ___________ for powerful numerical method.
    - (a) Initial Value
    - (b) Final Value
    - (c) Initial Starting Value
    - (d) Middle Value
12. Find (x_0, y_0) given that $y' = x + y, y(0) = 2$ using Taylor's formula
    - (a) $(1, 2)$
    - (b) $(2, 1)$
    - (c) $(0, 2)$
    - (d) $(2, 0)$
13. Which method requires prior calculation of higher derivatives?
    - (a) Taylor's Method
    - (b) Euler Method
    - (c) Adam's P-C Method
    - (d) Newton's Method
14. Which of the following method does not require starting values

(a) Euler's Method
(b) Adam's P-C Method
(c) Multistep Method
(d) All the above

15. $y_{n+1} = y_n + hf(x_n, y_n)$ is the iterative formula for

(a) Euler's Method
(b) Adam's P-C Method
(c) Modified Euler's Method
(d) Picard Method

16. Which of the following formula is a particular case of Runge-Kutta formula of second order?

(a) Taylor's Series
(b) Picard's Formula
(c) Modified Euler's
(d) Adam's P-C Method

17. Using Euler's method $\frac{dy}{dx} = \frac{y-2x}{y}, \quad y(0)=1$. The value of $y(0.1)$ is

(a) 1.1182
(b) 1.1818
(c) 1.1285
(d) 2.2356

18. Various types of Runge-Kutta methods are classified according to their

(a) Degree
(b) Order
(c) Rank
(d) Both (a) and (b)

19. The methods which do not require the calculation of higher order derivatives is

(a) Taylor's Method
(b) R-K Method
(c) Both (a) and (b)
(d) None of these

20. Runge-Kutta method is better than Taylor's method because

(a) It does not require prior calculation of higher derivatives
(b) It requires at most first order derivatives
(c) It does require prior calculation of higher derivatives
(d) All of the above

21. To solve the ODE $3\frac{dy}{dx} + xy^2 = \sin x, \quad y(0)=5$ by Runge-Kutta 4th order method, you need to rewrite the equation as

(a) $\frac{dy}{dx} = \sin x - xy^2, \quad y(0)=5$
(b) $\frac{dy}{dx} = \frac{1}{3}\left(\sin x - xy^2\right), \quad y(0)=5$
(c) $\frac{dy}{dx} = \frac{1}{3}\left(-\cos x - xy^3\right), \quad y(0)=5$
(d) $\frac{dy}{dx} = \frac{1}{3}\sin x - xy^2, \quad y(0)=5$

22. Which of the following method is called step by step method

(a) Taylor's Method
(b) R-K Method
(c) Adam's Method
(d) Newton's Method

23. $y_1 = y_0 + \frac{h}{24}\left(9 f_1 + 19 f_0 - 5 f_{-1} + f_{-2}\right)$ is the formula for

(a) Adam's Predictor
(b) Adam's Corrector
(c) R-K Method
(d) Modified Euler's Corrector

24. Given $3\frac{dy}{dx} + 5y^2 = \sin x,\ y(0.3) = 5$ and using a step size of $h = 0.3$, the value of $y(0.9)$ using Euler's method is most nearly

(a) – 35.318
(b) – 36.458
(c) – 658.91
(d) – 669.05

25. Given $3\frac{dy}{dx} + \sqrt{y} = e^{0.1x},\ y(0.3) = 5$ and using a step size of $h = 0.3$, the best estimate of $y(0.9)$ using Euler's method is most nearly

(a) – 0.37319
(b) – 0.36288
(c) – 0.35381
(d) – 0.34341

26. The velocity (m/s) of a body is given as a function of time (seconds) by $v(t) = 200\ln(1+t) - t,\ t \geq 0$

Using Euler's method with a step size of 5 seconds, the distance in meters traveled by the body from $t = 2$ to $t = 12$ seconds is most nearly

(a) 3133.1
(b) 3939.7
(c) 5638.0
(d) 39397

27. Given $3\frac{dy}{dx} + 5y^2 = \sin x,\ y(0.3) = 5$ and using a step size of $h = 0.3$, the value of $y(0.9)$ using the Runge-Kutta 2nd order method is most nearly

(a) –4297.4
(b) –4936.7
(c) -0.21336×10^{14}
(d) -0.24489×10^{14}

28. Given $3\frac{dy}{dx} + 5\sqrt{y} = e^{0.1x},\ y(0.3) = 5$ and using a step size of $h = 0.3$, the best estimate of $y(0.9)$ using the Runge-Kutta 2nd order midpoint method most nearly is

(a) –2.2473
(b) –2.2543
(c) –2.6188
(d) –3.2045

29. Let $P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

(a) – 30
(b) – 15
(c) – 5
(d) $-\frac{5}{6}$

30. The velocity (m/s) of a body is given as a function of time (seconds) by $v(t) = 200\ln(1+t) - t,\ t \geq 0$ using the Runge-Kutta 2nd order method with a step size of 5 seconds, the distance in meters traveled

by the body from $t = 2$ to $t = 12$ seconds is estimated most nearly as

(a) 3904.9
(b) 3939.7
(c) 6556.3
(d) 39397

31. Given $3\frac{dy}{dx} + y^2 = e^x, y(0.3) = 5$, and using a step size of $h = 0.3$, the best estimate of $y(0.9)$ using Runge-Kutta 4th order method is most nearly

(a) -1.6604
(b) -1.1785
(c) -0.45831
(d) 2.7270

32. The velocity (m/s) of a parachutist is given as a function of time (seconds) by $v(t) = 55.8\tanh(0.17t),\ t \geq 0$ using Runge-Kutta 4th order method with a step size of 5 seconds, the distance in meters traveled by the body from $t = 2$ to $t = 12$ seconds is estimated most nearly as

(a) 341.43
(b) 428.97
(c) 429.05
(d) 703.50

33. Given $\frac{d^2y}{dx^2} = 6x - 0.5x^2$, $y(0) = 0$, $y(12) = 0$, if one was using shooting method with Euler's method with a step size of $h = 4$, and an assumed value of $\frac{dy}{dx}(0) = 20$, then the estimated value of $y(12)$ in the first iteration most nearly is

(a) 60.00
(b) 496.0
(c) 1088
(d) 1102

34. The accuracy of the Euler method can be improved by using ________________

(a) smaller step size
(b) larger step size
(c) initial value
(d) final value

35. Varies types of Runge-Kutta methods are classified according to their

(a) Degree
(b) Order
(c) Rank
(d) Both (a) and (b)

SECTION THREE

ANSWER ALL questions in this section in the answer booklet

1\. (a) Rewrite the following differential equation as a set of first order differential equations, $3\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=e^{-x},\ y(0)=5,\ y'(0)=7$ and hence use fourth order Runge-Kutta's method. Compute $y(1)$ with $h=0.25$. Leave your answers in 4 decimal places.

(b) Given $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=e^{-t}, y(0)=1, \frac{dy}{dt}(0)=2$, find by Euler's method

(i) $y(0.75)$

(ii) the absolute relative true error for part *(i)*, if $y(0.75)\big|_{exact}=1.668$

Use a step size of $h=0.25$. Leave your answers in 4 decimal places.

GOOD LUCK!!!

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