UNIVERSITY OF MINES AND TECHNOLOGY – TARKWA

SECOND SEMESTER EXAMINATION, MAY, 2018

COURSE NO: MA 374

COURSE NAME: **ABSTRACT ALGEBRA**

CLASS: MA III TIME: 3 HOURS

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS OF YOUR CHOICE.
ALL QUESTIONS CARRY EQUAL MARKS

Q1. (a) Let X and Y be groups with identities e and I respectively. Let $f : X \to Y$ be a homomorphism. Prove that $f\left(a^{-1}\right) = f\left(a\right)^{-1}, \ \forall a \in X$

(b). If G is a dihedral group of order eight. Prove that the centre C(G) of the dihedral group is

i. $\left\{ I, \ a^2 \right\}$ and

ii. the quotient group $\dfrac{G}{C(G)}$ is Klein's four group.

(c). Suppose that R is a commutative ring with unity. $\forall \ \alpha \in R$ and f $\in R[x]$, Prove that these

two statements are equivalent

i. f $= g(x-\alpha)^2$

ii. f (α) = 0 and Df (α) = 0, where Df is the derivative of f .

Q2. (a). Explain why a nonempty set Q defined on the operation $*$ is a group.

(b). Let G be a group and H a normal subgroup of G. For every pair $H_x, H_y \in \dfrac{G}{H}$

define $H_x \bullet H_y = H_{xy}$. Prove that $\left(\dfrac{G}{H}, \bullet\right)$ is a group under the operation $\bullet$

(c). Given that Z and R are the sets of integers and real numbers define on the multiplicative operation $*$, explain whether or not the following are groups.

i. $(Z,*)$ ii. $(R,*)$

iii. Given that $\boldsymbol{H} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 9 & 4 & 1 & 5 & 6 & 2 & 7 & 8 \end{pmatrix}$. Find the cycle of $\boldsymbol{H}$ determined by 1, 2, 4, 5 and the orbit of 2 under $\boldsymbol{H}$.

Q3. (a). State Lagrange's theorem

(b). Suppose $(S,*)$ has an identity element *e* for $*$. If $f: S \to S^1$ is an isomorphism of $(S,*)$ with $(S^1,*^1)$ and $f(e)$ is an identity element for the binary operation $*^1$ on S^1. Prove that $f(e)*s^1 = s^1 *^1 f(e) = s^1$.

(c). Given that X = { 1, 2, 3}. Determine the symmetric group of degree three S_3 of the set X.

(d). In S_3 all permutations of the set X = (1, 2, 3) are;

$$a = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, b = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, c = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$d = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \quad e = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

Copy and complete the table using the permutation above

	a	b	c	d	e	f
a						
b						
c						
d						
e						
f						

Q4. (a) Let P(x) be a polynomial in K[x] and T[x] $\in$K[x] such that P(x) $\neq 0$. If there exist q(x), r (x) $\in$K[x] such that the deg r (x) < deg P(x), then T(x) = q(x)p(x) + r(x) Prove.

(b). *INDICATE WHETHER THE FOLLOWING STATEMENTS ARE TRUE OR FALSE.*

i. Every group of order less than or equal to four is cyclic. (*TRUE/ FALSE*)

ii. All generators of Z_{10} are prime. (*TRUE / FALSE*)

iii. Every cyclic group is Abelian. (*TRUE / FALSE)*

iv. Every Abelian group is cyclic. (*TRUE / FALSE*)

v. There is at least one Abelian group in every finite order greater than zero (*TRUE / FALSE*)

vi. A_3 is a cyclic group. (*TRUE / FALSE*)

(c). A homomorphism $f : X \to Y$ is injective if and only if $Ker f = (e)$. Prove.

Course Examiner: Dr L. Brew