UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV. – DEC. 2018

COURSE NO: MA 373 Unihubgh.com

COURSE NAME: LINEAR PARTIAL DIFFERENTIAL EQUATIONS

CLASS: MA III **TIME:** 2 HRS 45 MIN

Name: ______________________________________ Index Number:

SECTION A: Answer all Questions in this section

Solve and write down your answer in the answer booklet provided.

1. By eliminating a and b from $(x+a)+(y+b)=z$ the partial differential equation formed is
   a. $z=p+q$
   b. $z=(p-q-1)$
   c. $z=(pq+1)$
   d. $z=pq$

2. The equation $\frac{\partial^2 z}{\partial x^2}+5xy\left(\frac{\partial z}{\partial x}\right)^3+\frac{\partial z}{\partial y}=5$ is order ⋯⋯ and degree ....
   a. (1,2)
   b. (1,1)
   c. (2,1)
   d. None of the above

3. The nature of the partial differential equation $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial y\partial x}-\frac{\partial^2 z}{\partial y^2}=0$ is
   a. Parabolic
   b. Hyperbolic
   c. Elliptic
   d. Neumannr

4. The general solution of the equation $xp-yq=y^2-x^2$ is
   a. $\emptyset(xy,x^2+y^2-2u)=0$
   b. $\emptyset(xy,x^2-y^2-2u)=0$
   c. $\emptyset(xy,x^2+y^2+2u)=0$
   d. $\emptyset(xy,x^2-y^2+2u)=0.$

5. The multiplies of the equation $x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$ is
   a. $\left(\frac{1}{x},\frac{1}{y},\frac{1}{z}\right),(x,y,-1)$
   b. $(x,y,z),(x,y,-1)$
   c. $\left(\frac{1}{x},\frac{1}{y},\frac{1}{z}\right),(x,y,1)$
   d. None of the above

6. . If $u=x^2+4t^2$ is a solution of $c^2\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial x^2}$, then $c=\cdots\cdots\cdots$
   a. 1
   b. 2
   c. 0
   d. 3

7. The general solution of $u_{xx}=x^2y$ is
   a. $u=\frac{1}{12}x^4y-xf(y)-\emptyset(y)$
   b. $u=\frac{1}{12}x^4+xf(y)+\emptyset(y)$
   c. $u=\frac{1}{12}x^4y+xf(y)+\emptyset(y)$
   d. $u=\frac{1}{12}x^4y+xf(x)+\emptyset(x)$

8. The PDE formed by eliminating arbitrary function f from the relation $z = f(\frac{xy}{z}, z) = 0$ is

a. $px - qy = 0$

b. $py - qx = 0$

c. $py + qx = 0$

d. $px + qy = 0$

9. $u = -x^2 + \frac{y^2}{2}$ is a solution of ⋯⋯

a. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

b. $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$

c. $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial y^2} = 0$

d. $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial y^2} = 0$

10. All the following are methods of solving second order linear partial differential equations except

a. Separation of variables

b. Lagrange's method

c. Convolution method

d. Green function

11. The general solution of the PDE $3u_{xx} + 4u_{xy} + u_{yy} = 0$ is

a. $u = F\left(x + \frac{1}{3}y\right) + G(x - y)$

b. $u = F\left(x - \frac{1}{3}y\right) + G(x + y)$

c. $u = f\left(x - \frac{1}{3}y\right) + g(x - y)$

d. $u = F\left(x + \frac{1}{3}y\right) + G(x - y)$

12. A parabolic equation of the form $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}$ is known as a ⋯

a. 2-D Wave Equation

b. 3-D Heat Equation

c. 2-D Fourier Equation

d. 3-D Poisson equation

13. If $f(x)$ satisfies Dirichlet's condition, then the Fourier series $f(x)$ is convergent and its sum is

a. $\frac{1}{2}[f(x+0)]$

b. $\frac{1}{2}[f(x+0) - f(x-0)]$

c. $\frac{1}{2}[f(x+0) + f(x-0)]$

d. $\frac{1}{2}[f(x-0)]$

14. Which of the following is also known as the complexified version of diffusion equation?

a. $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$

b. $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$

c. $\frac{\partial^2 u}{\partial x^2} + i\frac{\partial u}{\partial t} = 0$

d. $\frac{\partial^2 u}{\partial x^2} = i\frac{\partial u}{\partial t}$

15. Another name for Schrödinger's equation is

a. Wave equation

b. Laplace equation

c. Diffusion equation

d. Gordon's equation

16. . If $f(x, y, a, b) = 0$, then the P.D.E formed by eliminating the arbitrary constants a and b is of

a. First Order

b. Second Order

c. Third Order

d. Fourth Order

17. The general solution $U = F(x + qy) + (rx + sy)G(x + qy)$ if the homogeneous first order PDE equation is

a. Exact

b. Elliptic

c. Parabolic

d. Hyperbolic

18. The solution of $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$ is

a. $\sqrt{x} - \sqrt{y} = f(\sqrt{x} - \sqrt{y})$

b. $\sqrt{x} - \sqrt{y} = f(x - y)$

c. $x - y = f(x - y)$

d. $x - y = f(\sqrt{x} - \sqrt{y})$

19. The partial differential equation $z = \frac{\partial^2 z}{\partial x^2}$ gets reduced to the form

a. Linear Homogeneous D.E

b. First Order PDE

c. First Order Linear PDE

d. None of these

20. All the following are classification of PDE's except

a. Order

b. Linearity

c. Coefficients

d. Homogeneity

SECTION B: Answer <u>Two</u> Questions from this section

Q1. (a) Form a Partial Differential equation by eliminating the arbitrary function *f* from

$$z = y^2 + 2f\left(\frac{1}{x} + logy\right)$$

(b) Solve by Lagrange's method the linear Partial differential equation $xp - yq = x^2 + y^2$

(c) Find the general solution for the non- homogeneous Partial Differential equation

$$\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} = x(2y + 3x)$$

Subject to the boundary conditions $u = 2x$ and $\frac{\partial u}{\partial y} = -6$ for $y = 0$.

Q2(a). Find the full-range Fourier Series expansion of $f(x)$ in the interval $-3 \leq x \leq 3$, where

$$f(x) = \begin{cases} 0 \;, & -3 \leq x < 0 \\ (2 + x), & 0 \leq x \leq 3 \end{cases}$$

(b) Solve the Cauchy Problem for the wave equation below using d'Alembert solution approach:

$u_{xx} = 9u_{tt}$ for $-\infty < x < \infty, t > 0$, with $u(x.0) = cosx$ and $u_t(x, 0) = sin2x$.

Q3. (a) Use the method of separation of variables to solve $\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial x} + u$, given that

$u(x, 0) = 6e^{-3x}$

(b) A bar of length 2m is fully insulated along its sides. It is initially at a uniform temperature of 10^0C and at t=0. The ends are plunged into ice and maintained at a temperature of 0^0C. Determine an expression for the temperature at a point P, a distance $x\ meter$ from one end at any subsequent time t seconds after t=0.

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