UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

SECOND SEMESTER EXAMINATIONS, MAY 2018

COURSE NO: MA 372
COURSE NAME: **SPECIAL MATHEMATICAL FUNCTIONS**
CLASS: MA III **TIME:** 3 HOURS

Name**:** ____Index Number: _______________

ANSWER ANY THREE QUESTIONS.

EACH QUESTION MUST BE ON A FRESH PAGE.

Q1(a). State Leibnitz's theorem of the nth differentiation of a product.

(b).A differential equation given by $4xy'' + 2y' + y = 0$ has a regular singular point about $x = 0$. Assuming an infinite solution of the form $y = \sum_{r=0}^{\infty} a_r x^{m+r}$ obtain the roots of the indicial equation. Also obtain the recurrence relation and show that, the complete solution to the differential equation in the neighbourhood of $x = 0$ is given by $y = a_0 \cos\sqrt{x} + b_0 \sin\sqrt{x}$.

Q2(a). Define the Gamma and Beta functions

(b). Given that $\int_0^{\pi/2} (\cos\theta)^{2m-1} (\sin\theta)^{2n-1} d\theta = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}$, $\{m > 0,\ n > 0\}$. Evaluate the following if $\Gamma(0.75) = 1.2254$ and $\Gamma(0.25) = 3.6256$:

(i) $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$ (ii) $\int_0^{\pi/2} \sin^5\theta \cos^4\theta\, d\theta$

(c). Using the substitution $y = \frac{x(1+a)}{a+x}$, show that

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}} dx = \frac{\Gamma(m)\Gamma(n)}{a^n (1+a)^m \Gamma(m+n)} \quad \text{where} \quad m > 0, \quad n > 0$$

Q3(a). Using the generating function $Q(x, y) = (1 - 2xy + h^2)^{-1/2}, |h| < 1$, obtain the first five terms

of Legendre's polynomials.

(b). Given that $p(x,t)=\left(1-2xt+t^2\right)^{-1/2}, |t| \prec 1,$ show that $t\frac{\partial}{\partial t}\{tp(x,t)\}=(1-tx)\frac{\partial p}{\partial x}$ and then substitute $p(x,t)=\sum_{n=0}^{\infty} p_n(x)t^n$ to prove that for any positive integer n

$$np_{n-1}(x)-p_n^{/}(x)+xp_{n-1}^{/}(x)=0$$

Q4(a). Define the factorial function $(a)_m$ and show that if m is a positive integer then

$$\Gamma(a+m)=(a)_m\Gamma(a)$$

(b). Prove that $(-n)_k=\frac{(-1)^k n!}{(n-k)!}$ where $(a)_k$ denotes the factorial function

(c). If $y=\frac{\sin x}{1-x^2}$, show that $\left(1-x^2\right)y^{//}-4xy^{/}-\left(1+x^2\right)y=0$ and differentiate this result n times using Leibnitz formula for the differentiation of the product.

***Examiner: Peter Kwesi Nyarko** / Henry Otoo*