UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, MAY 2018

COURSE NO : **MA 174**
COURSE NAME: **HIGHER LINEAR ALGEBRA**
CLASS : MA **I** TIME: **2 HOURS**

Name: _____ Index Number: _____

(Answer **two** Question **only**)

1. a) Suppose $\boldsymbol{u}$ and $\boldsymbol{v}$ are two vectors in R^n and θ is the angle between them. Prove that

$$u \cdot v = \|u\|\|v\|\cos(\theta)$$

b) Suppose $\boldsymbol{u}$ and $\boldsymbol{v}$ are two vectors in R and c is a scalar, prove that

i) $\|\boldsymbol{u}\| \geq 0$

ii) $\|c\boldsymbol{u}\| = |c|\|\boldsymbol{u}\|$

iii) $\|\boldsymbol{u}+\boldsymbol{v}\| \leq \|\boldsymbol{u}\| + \|\boldsymbol{v}\|$

c) A subspace of P_4 is a vector space of polynomials with degree 4. Let subspace W be defined as $\{p(x) | p \in P_4, p(2) = 0\}$. Is W a subspace?

2. a) Define Basis.

b) Let $S = \{v_1, v_2, v_3\}$ where $v_1 = (1,0,1)$, $v_2 = (1,1,0)$, and $v_3 = (1,2,-1)$.

c) Suppose $W = \{x, x_2, x_3\}$ where

$x_1 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$, $x_2 = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$, and $x_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ is a basis for the subspace W of $\mathbb{R}^3$.

Using Gram-Schmidt Orthogonalisation process, find the orthogonal basis for W

3. a) Consider the inner product $C[0,1]$. If $f(x) = 5x$ and $g(x) = 7x^3 - 3$, find

i) $\|f\|$

ii) $\langle f, g \rangle$

iii) $d(f, g)$

b) i) Simplify $\langle 2t\boldsymbol{u}+t\boldsymbol{v}, 3t^2\boldsymbol{u}-t\boldsymbol{v}\rangle$ where t is a scalar.

ii) Given that $t=-2$, $\|\boldsymbol{u}\|=-3$, $\langle \boldsymbol{u},\boldsymbol{v}\rangle = 12$ and $\|\boldsymbol{v}\|=5$, evaluate the solution in i).

c) Assume that $S=\{\boldsymbol{u}_1,\boldsymbol{u}_2\}$ where $\boldsymbol{u}_1=\begin{pmatrix}-2\\4\end{pmatrix}$ *and* $\boldsymbol{u}_2=\begin{pmatrix}3\\3\end{pmatrix}$. Is the vector $\boldsymbol{w}=\begin{pmatrix}10\\-2\end{pmatrix}$ *in* span$(\boldsymbol{u}_1,\boldsymbol{u}_2)$?

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