



UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA  
FIRST SEMESTER EXAMINATIONS, APR-MAY, 2023

COURSE NO : MA 375  
COURSE NAME : MATHEMATICAL MODELLING  
CLASS : MA III TIME: 3 HOURS

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Name: \_\_\_\_\_ Index Number: \_\_\_\_\_

**INSTRUCTION:** Answer ALL Questions. The entire exam has 60 points on it, each worth the indicated number of points Use your time to achieve the maximum number of points possible.

1. Read the whole exam before beginning.
2. Make sure you have all pages.
3. Organization and neatness count.
4. Justify your answers.

- ✓ 1. a) What is meant by the term 'Mathematical Model'? 2 marks  
b) State and explain four (4) benefits of using mathematical models. 8 marks

- ✓ 2. Consider the following system of linear differential equations:

$$\begin{cases} \dot{x}_1 = \alpha x_1 + \gamma x_2 + c_1 \\ \dot{x}_2 = \beta x_1 + \alpha x_2 + c_2 \end{cases}$$

where  $\alpha, \beta, \gamma, c_1, c_2$  are real constants.

- a) Write the equations in the standard form  $\dot{x} = Ax + b$ . 2 marks  
b) Compute the eigenvalues and eigenvectors of matrix  $A$ . 4 marks  
c) If  $\alpha = 1, \beta = -2, \gamma = 2, c_1 = -3$  and  $c_2 = 1$  classify the fixed point. 4 marks  
d) Find the general solution of the equation for values  $\alpha, \beta, \gamma, c_1, c_2$  of given in (c) 3 marks
- ✓ 4. For each of the following systems of equations, find the eigenvalues and corresponding eigenvectors. Use the eigenvalues to classify each critical point as center, spiral, saddle, or node. Are the critical points stable or unstable (justify your reasoning), Hence find the general solution, and sketch a phase portrait:
- a)  $x' = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} x$  5 marks  
b)  $x' = \begin{pmatrix} 5 & 0 \\ 2 & -1 \end{pmatrix} x$  5 marks  
c)  $x' = \begin{pmatrix} -1 & -4 \\ 2 & 3 \end{pmatrix} x$  5 marks  
d)  $x' = \begin{pmatrix} 4 & -1 \\ 1 & 6 \end{pmatrix} x$  5 marks

✓ 3. Given the system of two competing species model

$$\begin{cases} \dot{x} = x(3 - x - 2y) \\ \dot{y} = y(4 - 3x - y) \end{cases}$$

8 marks

- a) Find the stationary points of the system.
- b) Compute the linearization of the system around the nontrivial stationary points and classify them accordingly.

9 marks

GOOD LUCK!!!

Dr E. N. Wiah

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