



UNIVERSITY OF MINES AND TECHNOLOGY,
FIRST SEMESTER EXAMINATIONS, APRIL 2022

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COURSE NO : MA 375
COURSE NAME : MATHEMATICAL MODELLING
CLASS : MA III TIME: 3 HOURS

Name: _____ Index Number: _____

INSTRUCTIONS

There are three (3) questions on this paper. Answer ALL questions in the answer booklet provided. All questions carry equal marks. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations.

Question 1

Let $A = \begin{bmatrix} \beta & -2 \\ 2 & 1 \end{bmatrix}$, and consider the homogeneous linear system $\dot{u} = Au$ with parameter β .

- Explain how the system changes as the parameter β changes. (HINT: Use the Tr-Det diagram)
- Choose a convenient value for β for each of the types above, solve, and sketch the trajectories in the vicinity of the critical point.

Question 2

Suppose there is an interaction between two species H and P . We wish to model the $H - P$ interaction given the following assumptions:

Species H assumptions:

- H grow exponentially in the absence of species P .
- The presence of species P depresses species H growth at a rate jointly proportional to the H and P populations.

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- Species P grow exponentially in the absence of species H .
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- Formulate a mathematical model with continuous time for this situation. Label all the variables and state any assumption that you make about the parameters.
- Find the steady-state(s) of the system
- Determine the stability of the steady-state(s)
- Characterize the steady-state(s) as nodes, spirals or saddle points

Question 3

The system of differential equation that models the system is

$$\begin{cases} \frac{dx}{dt} = x(1-x) - \frac{1}{2}xy \\ \frac{dy}{dt} = y(1-y) \end{cases}$$

- Compute the equilibrium points of the system.
- Determine the stability of the equilibrium point and classify each of the steady state.
- Find the eigenvalues of the coefficient matrix of the linearized system.
- Sketch the phase portraits around each equilibrium point.

GOOD LUCK!!!

Examiner: Dr E. N. Wiah