



UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV. – DEC. 2018

COURSE NO: GM/GL/MN/MR/PE/ES 153

COURSE NAME: LINEAR ALGEBRA AND TRIGONOMETRY

CLASS: GM/GL/MN/MR/PE/ES I

TIME: 3HOURS

Name: _____ Index Number: _____

Answer ALL QUESTIONS in SECTION A, and ANY TWO questions in SECTION B. Write only your answers for section A. Answer all questions in the answer booklet. All computations should be done in the answer booklet.

SECTION A

1. Find the reciprocal of the number “ i ”.
2. Find the argument of the complex number “ $-1 + i$ ”
3. If a complex number z lies on a circle of radius $1/2$ then the complex number $4z$ will lie on a circle of radius
4. If $Z_1 = 2 - i$, $Z_2 = 2 - 4i$ and $\frac{1}{Z_3} = \frac{1}{Z_1} + \frac{1}{Z_2}$. Find Z_3
5. Find the trace of the matrix A if

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

6. Find the rank of the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 8 & 9 & 4 \\ 2 & 1 & 2 \end{bmatrix}$

7. What is the size of the matrix product

$$\begin{bmatrix} 1 & -11 & 2 \\ 0 & 7 & 0 \\ -3 & 4 & 9 \\ 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & -2 & 4 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

8. What values of x satisfies the matrix equation

$$\begin{bmatrix} 0 & -2+x \\ 4 & 3x-2 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 2+2x & 7x+14 \end{bmatrix}^T$$

9. Solve for x and y in the following matrix equation:

$$\begin{bmatrix} 3 & -2 & 2 \\ 1 & 0 & -1 \end{bmatrix} + \begin{bmatrix} x-y & 3 & -2 \\ 4 & x & y \end{bmatrix} = \begin{bmatrix} 6 & 1 & 0 \\ 5 & 2x+5 & -9 \end{bmatrix}$$

10. Find the matrix product if possible

$$\begin{bmatrix} 2 & 0 & 2 \\ 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -2 & 0 \\ 5 & 1 \end{bmatrix}$$

11. Find the determinant of

$$\begin{bmatrix} 3 & 5 & 6 & 1 \\ 4 & 0 & 2 & 8 \\ 1 & 0 & 7 & 2 \\ 2 & 0 & 1 & 3 \end{bmatrix}$$

Given that $\underline{a} = 3\underline{i} - 4\underline{j} + \underline{k}$, $\underline{b} = 5\underline{j} + 2\underline{k}$, $\underline{c} = \underline{i} - \underline{j} - \underline{k}$, solve the questions in 11, 12 and 13.

12. The resultant of \underline{a} , \underline{b} , and \underline{c}

13. $2\underline{a} + 3\underline{b} - 4\underline{c}$

14. $|\underline{a} - \underline{b}|$

15. A point P in space has coordinates (3,2, -4). How far is P from the origin
16. What is the angle between vectors \underline{a} and \underline{b} given that $\underline{a} = 3\underline{i} + 2\underline{j} + \underline{k}$ and $\underline{b} = -\underline{i} + 4\underline{j} + 5\underline{k}$?
17. Given that $\tan\theta = \frac{7}{24}$, where θ is an acute angle, find $\sin\theta$ and $\cos\theta$
18. Evaluate the expression $\cos^4\theta - \sin^4\theta$ given that $\cos^2\theta - \sin^2\theta = \frac{3}{4}$
19. A ladder 13m long stands on horizontal ground leaning against a vertical wall. The ladder makes an angle of 50° with the ground. How far is the foot of the ladder from the wall?
20. A surveyor measures the angle of elevation of the top of a perpendicular building as 19° . He moves 120 m nearer the building and finds the angle of elevation is now 47° . Determine the height of the building.

SECTION B

Question 1

- a. Given that $z = \cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)$. Find the sum of the series

$$z + z^2 + z^3 + \dots + z^{20}$$
- b. Find the expansion of $\sin 5\theta$ using De-Moivre's theorem
- c. Given that $z_1 = 2i$, $z_2 = (i - 1)$, $z_3 = (\sqrt{3} + i)$, $z_4 = 1 + i$, simplify

$$z_1 z_2 + (\bar{z}_3)^3 + z_4 \bar{z}_4$$
- d. Find all the 4 fourth roots of $9 + 4i$
- e. Solve the system of linear equations using Gaussian elimination method

$$\begin{aligned} 4x_1 - 3x_2 + x_3 + x_4 &= -5 \\ x_1 + x_2 + 3x_3 - x_4 &= -4 \\ 5x_1 - 2x_2 - x_3 + 2x_4 &= 5 \\ x_1 + x_2 - 5x_3 + 8x_4 &= 30 \end{aligned}$$

Question 2

- a. A constant force of magnitude 5N in a direction $\underline{i} + 2\underline{j} - \underline{k}$, displaces a particle from the point (1,0,1) to the point (3,4, -1). Calculate the amount of work done by the force.
- b. Given two vectors \underline{a} and \underline{b} where $\underline{a} = 2\underline{i} - \underline{j} + \underline{k}$ and $\underline{b} = 3\underline{i} + 2\underline{j} - 4\underline{k}$, if λ , μ and ρ are the direction cosines, show that $\lambda^2 + \mu^2 + \rho^2 = 1$ for each of the vectors
- c. Determine the area of a parallelogram $ABCD$ whose position vectors are $\underline{a} = 3\underline{i} - 4\underline{j} + \underline{k}$
 $\underline{b} = \underline{i} + 2\underline{j} - \underline{k}$, $\underline{c} = 3\underline{i} + 4\underline{j} - 2\underline{k}$, $\underline{d} = 4\underline{i} + 3\underline{j} - \underline{k}$
- d. Find the eigenvalues and eigenvectors of

$$\begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$$

Question 3

- a. If $\sin \theta = \frac{3}{5}$, find the value of $\sec 2\theta$
- b. A ladder 5m long, leans against a vertical wall at an angle 70° to the ground. The ladder slips down the wall 2m. Find, correct to two decimal places, the new angle which the ladder makes with the ground and the distance the ladder slipped back on the ground from its original position.
- c. Given that $\sin^2 \theta = \frac{1}{3}$, evaluate $72 \cos 2\theta$
- d. Evaluate $\tan 2x$ if $\cos x = \frac{3}{5}$
- e. Find the determinant of

$$\begin{bmatrix} 2 & -1 & 3 & 0 \\ -3 & 1 & 0 & 4 \\ -2 & 1 & 4 & 1 \\ -1 & 3 & 0 & -2 \end{bmatrix}$$

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