UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

SECOND SEMESTER EXAMINATIONS, SEP/OCT 2022

COURSE NO: MA 378

COURSE NAME: COMPLEX ANALYSIS

CLASS: MA III **TIME: 3 HOURS**

Name: _____ Index Number: _____

Answer all Questions

Question 1

a. Prove that $|z+w| \leq |z|+|w|$

b. What is the difference between a single-value function and a multi-valued function?

c. Define the following

i. Complex function

ii. Analytic function

iii. Harmonic

iv. Harmonic Conjugate

d. If $f(z) = 2z^2 + \frac{1}{z}$, then $f(2+i)$ $\frac{37}{5} + 7i$

[20 Marks]

Question 2

a. State the sufficient condition for a function to be analytic and differentiable. $u_x = v_y$ $u_y = -v_x$

b. Show the function $u(x,y) = e^{2x}(x\cos 2y - y\sin 2y)$ is Harmonic and find its Harmonic conjugate.

c. Given $f(z) = z^2 + 3z$, find u, v and calculate the value of f at 1+3i.

[20 Marks]

Question 3

a. Prove that if $f(z) = u(x,y) + iv(x,y)$ is differentiable at the point $z_0 = x_0 + iy_0$, then the first partial derivatives of $u(x,y)$ and $v(x,y)$ exist at (x_0, y_0) and satisfy the Cauchy-Riemann equations.

b. Show that the function $f(z) = e^{-y}\cos x + ie^{-y}\sin x$ is differentiable for all z, and find its derivative.

c. Prove that for a given complex function $f(z) = \dfrac{f^{(n)}(z_0)(z-z_0)^n}{n!}$

d. Find the Power Series representation for $f(z) = \cos z$

$$\sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$$

[40 Marks]

Examiner: Dr H. Otoo

$e^{-y}(\cos x + i\sin x)$

cos z
Sin z
cos
sin z
cos z
sin z
cos z
n z

is a rule which associate each element in the domain A a single element in

$$\sum_{n=0}^{\infty} \frac{f^{(n)} f(z_0)(z-z_0)^n}{n!}$$

$$\sum \frac{f' f(z_0)(z-z_0)'}{1} \quad \frac{}{2}$$