



UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA
SECOND SEMESTER EXAMINATIONS, MAY 2018

COURSE NO: MN/MR/ GM/GL/PE/ES 150

COURSE NAME: CALCULUS

CLASS: MN/MR /GM/GL/PE/ES I

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TIME: 3 HOURS

Name: _____ Index Number: _____

INSTRUCTION: ATTEMPT ANY THREE (3) QUESTIONS. ALL CALCULATIONS MUST BE CLEARLY DONE IN THE ANSWER BOOKLET PROVIDED

Question 1

a. A rocket is shot vertically upwards with an initial velocity of **400ft/s**. Its height s after t seconds is $s = 400t - 16t^2$. How fast is the distance changing from the rocket to an observer on the ground **1800ft** away from the launching site when the rocket is still rising and **2400ft** above the ground?

b.

i. Determine

$$\int \frac{x^2}{(x-2)(x^2+1)} dx$$

ii. Find the derivative of $f(x) = \ln(\cos x) - \frac{1}{3}\sin^3 x$

c.

i. Given that $z = \frac{x^3-y^3}{xy}$, show that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$

ii. Find the solution to the initial value problem $ty' + 2y = t^2 - t + 1$, given that $y(1) = \frac{1}{2}$

Question 2

- a.
- Find the equation of the tangent line to the “asteroid” curve defined implicitly by the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ at the point $(\sqrt{27}, 1)$
 - Find $\frac{dy}{dx}$ given that $x^2 + y^2 = \tan^{-1}(x^3 + 3y)$
 - Find the Laplace transform of $f(t) = e^{at}$ where a is a constant.
- b. Evaluate the following
- $\int_0^{\infty} t^3 e^{-t} dt$
 - $\int_0^1 x^4(1-x)^3 dx$
- c.
- Determine $\int (\sin^2 x \cos^3 x - \sin 6x \cos 2x) dx$
 - Given that $f(x, y) = xe^y - \sin\left(\frac{x}{y}\right) + x^3y^2$, show that $f_{xy} = f_{yx}$

Question 3

- a.
- A rectangular sheet of metal having dimensions **20cm** by **12cm** has squares removed from each of the four corners and the sides bent upwards to form an open box. Determine the maximum possible volume of the box.
 - Evaluate $\int (6\sin 5x \sin 3x - \cos^5 x) dx$
- b.
- Find the solution to the differential equation $(6x^2 - y + 3)dx + (3y^2 - x - 2)dy = 0$
 - Evaluate $\int_0^{\infty} x^n e^{-2x} dx$
- c.
- If $w(x, y, z) = xz + e^{y^2z} + \sqrt{xy^2z^3}$, calculate all the first order partial derivatives of w .
 - Find the Laplace transform of $f(t) = 2e^{-t} + t$

Question 4

a. Determine the area of the largest piece of rectangular ground that can be enclosed by **100m** of fencing, if part of an existing straight wall is used as one side.

b. Evaluate the following

i. $\int_0^{2\pi} \sin x \, dx$

ii. $\int (\ln x + \cos^{-1} x) \, dx$

iii. $\int_0^2 \frac{x^2}{\sqrt{2-x}} \, dx$

iv. $\frac{\Gamma(\frac{5}{2})}{\Gamma(\frac{1}{2})}$

c.

i. Solve the differential equation $e^y dx + (2y + xe^y) dy = 0$

ii. Find the Laplace transform of $F(t) = te^{4t}$

Examiners: **K. Agyarko /P. Boye**