

SC4011

WASSCE 2020

FURTHER MATHEMATICS/
MATHEMATICS (ELECTIVE)1

Objective Test

1½ hours

1

THE WEST AFRICAN EXAMINATIONS COUNCIL

West African Senior School Certificate Examination
for School Candidates

SC 2020

FURTHER MATHEMATICS/ MATHEMATICS (ELECTIVE) 1

1½ hours

OBJECTIVE TEST

[40 marks]

Do not open this booklet until you are told to do so. While you are waiting, read and observe the following instructions carefully. Write your name and index number in the spaces provided above.

Answer all the questions on your Objective Test answer sheet.

- Use 2B pencil throughout.
- On the pre-printed answer sheet, check that the following details are correctly printed:
 - In the space marked *Name*, check your **surname** followed by your **other names**.
 - In the spaces marked *Examination, Year, Subject* and *Paper*, check 'WASSCE', 'SC 2020', 'FURTHER MATHEMATICS/ MATHEMATICS (ELECTIVE)', and '1' in that order.
 - In the box marked *Index Number*, your **index number** has been printed vertically in the spaces on the left-hand side, and each numbered space has been shaded in line with each digit. **Reshade** each of the shaded spaces.
 - In the box marked *Subject Code*, the digits 40112 are printed vertically in the spaces on the left-hand side. **Reshade** the corresponding numbered spaces as you did for your index number.
- An example is given below. This is for a male candidate whose *name* is Elliot Kofi ABANGA. His *index number* is 7102143958 and he is offering *Further Mathematics/Mathematics (Elective) 1*.

THE WEST AFRICAN EXAMINATIONS COUNCIL

ANSWER SHEET

PRINTED IN BLOCK LETTERS.	ABANGA ELLIOT KOFI	GHA
Name:		
Examination:	WASSCE	Year: SC 2020
Subject:	FURTHER MATHS. / MATHS. (ELECTIVE)	Paper: 1

INSTRUCTIONS TO CANDIDATES

- Use grade 2B pencil throughout.
- Answer each question by choosing one letter and shading it like this: A B C D E
- Erase completely any answer you wish to change.
- Leave extra spaces blank if the answer spaces provided are more than you need.
- Do not make any markings across the heavy black marks at the right hand edge of your answer sheet.

INDEX NUMBER										SUBJECT CODE											
7	0	1	2	3	4	5	6	7	8	9	4	0	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	
0	1	2	3	4	5	6	7	8	9	1	0	1	2	3	4	5	6	7	8	9	
2	0	1	2	3	4	5	6	7	8	9	1	0	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8	9	1	0	1	2	3	4	5	6	7	8	9
4	0	1	2	3	4	5	6	7	8	9	2	0	1	2	3	4	5	6	7	8	9
3	0	1	2	3	4	5	6	7	8	9	For Supervisors only If candidate is absent shade this space. <input type="checkbox"/>										
9	0	1	2	3	4	5	6	7	8	9											
5	0	1	2	3	4	5	6	7	8	9											
8	0	1	2	3	4	5	6	7	8	9											

Answer **all** the questions.

Each question is followed by four options lettered A to D. Find the **correct** option for each question and shade in **pencil** on your answer sheet, the answer space which bears the same letter as the option you have chosen. Give only **one** answer to **each** question. An example is given below.

The ages, in years, of four boys are 10, 12, 14 and 16. What is the mean age of the boys?

- A. 12 years
- B. $12\frac{1}{2}$ years
- C. 13 years
- D. $13\frac{1}{2}$ years

The correct answer is 13 years, which is lettered C, and therefore answer space C would be shaded.

A

B

C

D

Think carefully before you shade the answer spaces; erase completely any answers you wish to change.

Do all rough work on this question paper.

Now answer the following questions:

1. A binary operation $*$ is defined on the set of real numbers, R , by $x * y = x^2 - y^2 + xy$, where $x, y \in R$. Evaluate $(\sqrt{3}) * (\sqrt{2})$.

- A. $1 + \sqrt{6}$
- B. $\sqrt{6}$
- C. $\sqrt{6} - 1$
- D. $1 - \sqrt{6}$

2. Find the inverse of $\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$.

- A. $\begin{pmatrix} 5 & 1 \\ 2 & 3 \end{pmatrix}$
- B. $\begin{pmatrix} -5 & 2 \\ -1 & 3 \end{pmatrix}$
- C. $\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$
- D. $\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$

3. If $\cos x = -0.7133$, find the values of x between 0° and 360° .

- A. $135.5^\circ, 224.5^\circ$.
- B. $135.5^\circ, 213.5^\circ$.
- C. $123.5^\circ, 190.5^\circ$.
- D. $44.5^\circ, 224.5^\circ$.

4. If $\int_0^3 (px^2 + 16)dx = 129$, find the value of p .

- A. 6
B. 7
C. 8
D. 9

$$\left[\frac{px^3}{3} + 16x \right]_0^3 = 129$$

5. If $\begin{pmatrix} p+q & 1 \\ 0 & p-q \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 8 \end{pmatrix}$, find the values of p and q .

- A. $p = -5, q = 3$
B. $p = -5, q = -3$
C. $p = 5, q = -3$
D. $p = 5, q = 3$

6. Given that $x: R \rightarrow R$ is defined by $x = \frac{y+1}{5-y}$, $y \in R$, find the domain of x .

- A. $\{y: y \in R, y \neq 7\}$
B. $\{y: y \in R, y \neq 5\}$
C. $\{y: y \in R, y \neq 1\}$
D. $\{y: y \in R, y \neq 0\}$

7. Simplify: $\frac{\sqrt{5} + 3}{4 - \sqrt{10}}$.

- A. $\frac{2\sqrt{5}}{3} - \frac{5\sqrt{2}}{6} - \frac{1\sqrt{10}}{2} + 2$
B. $\frac{2\sqrt{5}}{3} + \frac{5\sqrt{2}}{6} + \frac{1\sqrt{10}}{2} + 2$
C. $\frac{2\sqrt{5}}{3} + \frac{5\sqrt{2}}{6} + \frac{1\sqrt{10}}{2}$
D. $\frac{2\sqrt{5}}{3} + \frac{5\sqrt{2}}{6} + 2$

8. If $\frac{6x+k}{2x^2+7x-15} \equiv \frac{4}{x+5} - \frac{2}{2x-3}$, find the value of k .

- A. -25
B. -24
C. -22
D. -21

9. Differentiate $\frac{x}{x+1}$ with respect to x .

- A. $\frac{1}{(x+1)^2}$
B. $\frac{1-x}{(x+1)^2}$
C. $\frac{-1}{x+1}$
D. $\frac{1}{x+1}$

10. Given that $2x + 3y - 10 = 0$ and $3x = 2y - 11$, calculate the value of $(x - y)$.
- A. -5
 B. -3
 C. 3
 D. 5
11. Given that $f(x) = 2x$ and $g(x) = 3x^2$, find the values of x for which $f^{-1}(x) = g(x)$.
- A. $x = \frac{1}{6}$ or $x = \frac{2}{3}$
 B. $x = \frac{1}{12}$ or $x = \frac{2}{3}$
 C. $x = 0$ or $x = \frac{2}{3}$
 D. $x = 0$ or $x = \frac{1}{6}$
12. If $V = P \log_x(M + N)$, express N in terms of X, P, M and V .
- A. $N = X^{\frac{P}{V}} + M$
 B. $N = X^{\frac{V}{P}} + M$
 C. $N = X^{\frac{P}{V}} - M$
 D. $N = X^{\frac{V}{P}} - M$
13. Determine the coefficient of x^3 in the binomial expansion of $(1 + \frac{1}{2}x)^5$.
- A. $\frac{5}{2}$
 B. $\frac{5}{4}$
 C. $\frac{5}{6}$
 D. $\frac{5}{8}$
14. Given that $P = \{x: 1 \leq x \leq 6\}$ and $Q = \{x: 2 < x < 10\}$, where x are integers, find $n(P \cap Q)$.
- A. 10
 B. 8
 C. 6
 D. 4
15. If $\sin x = \frac{3}{5}$ and $\cos y = \frac{24}{25}$, where x and y are acute, find the value of $\cos(x + y)$.
- A. $\frac{7}{25}$
 B. $\frac{3}{5}$
 C. $\frac{24}{25}$
 D. $\frac{117}{125}$

16. Find the median of the numbers 9, 7, 5, 2, 12, 9, 9, 2, 10, 10 and 18.
- A. 11
B. 10
C. 9
D. 7
17. Calculate the probability that the product of two numbers selected at random with replacement from the set $\{-5, -2, 4, 8\}$ is positive.
- A. $\frac{1}{6}$
B. $\frac{1}{3}$
C. $\frac{1}{2}$
D. $\frac{2}{3}$
18. Find the angle between $\mathbf{i} + 5\mathbf{j}$ and $5\mathbf{i} - \mathbf{j}$.
- A. 90°
B. 60°
C. 45°
D. 0°
19. Given that $\mathbf{F} = 3\mathbf{i} - 12\mathbf{j}$, $\mathbf{R} = 7\mathbf{i} + 5\mathbf{j}$ and $\mathbf{N} = p\mathbf{i} + q\mathbf{j}$ are forces acting on a body, if the body is in equilibrium, find the values of p and q .
- A. $p = 10, q = 7$
B. $p = 10, q = -7$
C. $p = -10, q = -7$
D. $p = -10, q = 7$
20. A stone was dropped from the top of a building 40 m high. Find, correct to **one** decimal place, the time it took the stone to reach the ground. [Take $g = 9.8 \text{ m s}^{-2}$]
- A. 1.4 seconds
B. 2.6 seconds
C. 2.8 seconds
D. 2.9 seconds
21. In which of the following series can the formula $s = \frac{a}{1-r}$, where a is the first term and r is the common ratio, be used to find the sum of all the terms?
- A. $128 + 64 + 32 + 16 + \dots$
B. $\frac{4}{81} + \frac{2}{27} + \frac{1}{9} + \frac{1}{6} + \dots$
C. $\frac{1}{2} + 2\frac{1}{2} + 12\frac{1}{2} + 62\frac{1}{2} + \dots$
D. $4 + 8 + 16 + 32 + \dots$
22. If the binomial expansion of $(1 + 3x)^6$ is used to evaluate $(0.97)^6$, find the value of x .
- A. -0.03
B. -0.01
C. 0.01
D. 0.03

Turn over

23. Find the n^{th} term of the linear sequence (A.P.) $(5y + 1), (2y + 1), (1 - y), \dots$
- A. $8y - 3n + 1$
 B. $(8 - 3n)y + 1$
 C. $8y + 3n + 1$
 D. $(8 + 3n)y + 1$
24. A circle with centre $(5, -4)$ passes through the point $(5, 0)$. Find its equation.
- A. $x^2 + y^2 - 10x - 8y - 25 = 0$
 B. $x^2 + y^2 - 10x + 8y + 25 = 0$
 C. $x^2 + y^2 + 10x - 8y - 25 = 0$
 D. $x^2 + y^2 + 10x + 8y + 25 = 0$
25. Calculate, correct to **two** decimal places, the area enclosed by the line $3x - 5y + 4 = 0$ and the axes.
- A. 0.54 square units
 B. 0.53 square units
 C. 0.51 square units
 D. 0.50 square units
26. In how many ways can the letters of the word MEMBER be arranged?
- A. 90
 B. 180
 C. 360
 D. 720
27. Which of the following is **not** an equation of a circle?
- ~~A.~~ $x^2 - y^2 + 3x - 5y = 2$
 B. $5x^2 + 5y^2 - 16 = 0$
 C. $x^2 + y^2 - 5x + 4y = 0$
 D. $3x^2 + 3y^2 + 5x + 7y = 5$
28. A function f defined by $f: x \rightarrow x^2 + px + q$ is such that $f(3) = 6$ and $f'(3) = 0$. Find the value of q .
- A. 21
 B. 15
 C. -6
 D. -9
29. In what interval is the function $f: x \rightarrow 2x - x^2$ increasing?
- A. $0 < x < 1$
 B. $x > 1$
 C. $x < 1$
 D. $x = 1$
30. A force of 230 N acts in the direction 065° . Find its horizontal component.
- A. 97.2 N
 B. 118.5 N
 C. 197.2 N
 D. 208.5 N

31. Calculate the variance of $\sqrt{2}$, $(1 + \sqrt{2})$ and $(2 + \sqrt{2})$.
- A. 2
B. $\frac{2}{3}$
C. $\sqrt{\frac{2}{3}}$
D. 0
32. A three-digit odd number less than 500 is to be formed from 1, 2, 3, 4 and 5. If repetition of digits is allowed, in how many ways can this be done?
- A. 36
B. 60
C. 75
D. 125
33. The variables x and y are such that $y = 2x^3 - 2x^2 - 5x + 5$. Calculate the corresponding change in y when x changes from 2.00 to 2.05.
- A. 1.95
B. 1.48
C. 0.95
D. 0.58
34. A bag contains 5 red and 5 blue identical balls. Three balls are selected at random **without replacement**. Determine the probability of selecting balls alternating in colour.
- A. $\frac{1}{36}$
B. $\frac{5}{36}$
C. $\frac{5}{18}$
D. $\frac{7}{18}$
35. The distance (s) in metres covered by a particle in motion at any time, t seconds, is given by $s = 120t - 16t^2$. Find, in metres, the distance covered by the body before coming to rest.
- A. 225
B. 223
C. 222
D. 220
36. $P(3, 4)$ and $Q(-3, -4)$ are two points in a plane. Find the gradient of the line that is normal to the line PQ .
- A. $-\frac{4}{3}$
B. $-\frac{3}{4}$
C. $\frac{3}{4}$
D. $\frac{4}{3}$

Turn over

37. Find the unit vector in the direction opposite to the resultant of forces $\mathbf{F}_1 = (-2\mathbf{i} - 3\mathbf{j})$ and $\mathbf{F}_2 = (5\mathbf{i} - \mathbf{j})$.
- A. $\frac{1}{5}(3\mathbf{i} + 4\mathbf{j})$
B. $\frac{1}{5}(3\mathbf{i} - 4\mathbf{j})$
C. $\frac{1}{5}(-3\mathbf{i} + 4\mathbf{j})$
D. $\frac{1}{5}(-3\mathbf{i} - 4\mathbf{j})$
38. If the sum of the roots of $2x^2 + 5mx + n = 0$ is 5, find the value of m .
- A. 2.5
B. 2.0
C. -2.0
D. -2.5
39. If $\log_5\left(\frac{125x^3}{\sqrt[3]{y}}\right)$ is expressed in the form $p\log_5 x + q\log_5 y + k$, find the values of p , q and k respectively.
- A. $3, -\frac{1}{3}, 3$
B. $3, \frac{1}{3}, 3$
C. $-\frac{1}{3}, 3, 5$
D. $3, -\frac{1}{3}, 5$
40. Consider the statements:
 x : Birds fly
 y : The sky is blue
Which of the following statements can be represented as $x \Leftrightarrow y$?
- A. When the sky is blue, the birds flies.
B. Either the bird is flying or the sky is blue.
C. Birds fly if and only if the sky is blue.
D. When birds fly, the sky is blue.

END OF PAPER

2
SECTION A
[48 marks]

Answer all the questions in this section. All questions carry equal marks.

1. A binary operation Δ is defined on the set of real numbers, R , by $x \Delta y = x + y + 10$. Find the:
- (a) identity element;
- (b) inverses of 3 and -5 under Δ .

2. Evaluate $\int_2^4 \left(\frac{x^3 + 3}{x^2} \right) dx$.

3. (a) Two functions f and g are defined on the set of real numbers, R , by $f: x \rightarrow x^2 - 1$ and $g: x \rightarrow x + 2$. Find $f \circ g(-2)$.
- (b) A bus has 6 seats and there are 8 passengers. In how many ways can the bus be filled?

4. Express $\frac{1}{x^2 - 16}$ in partial fractions.

5. The table shows the marks scored by some students in a class test.

Marks	11 - 14	15 - 18	19 - 22	23 - 26	27 - 30	31 - 34	35 - 38
No. of Students	4	5	18	31	25	14	3

- (a) Draw a histogram for the distribution.
- (b) Use the histogram to estimate the modal score, correct to **one** decimal place.
6. A bag contains 10 black and 5 yellow identical balls. **Two** balls are picked at random from the bag one after the other **without** replacement. Calculate the probability that the are:
- (a) **both** black;
- (b) of the **same** colour.
7. Forces $F_1(24 \text{ N}, 120^\circ)$, $F_2(18 \text{ N}, 240^\circ)$ and $F_3(12 \text{ N}, 300^\circ)$ act at a point. Find, correct to **two** decimal places, the magnitude of their resultant force.
8. The vectors \mathbf{p} , \mathbf{q} and \mathbf{r} are mutually perpendicular with $|\mathbf{q}| = 3$ and $|\mathbf{r}| = \sqrt{5.4}$. If the vectors $\mathbf{X} = 3\mathbf{p} + 5\mathbf{q} + 7\mathbf{r}$ and $\mathbf{Y} = 2\mathbf{p} + 3\mathbf{q} - 5\mathbf{r}$ are perpendicular, find $|\mathbf{p}|$.

3
SECTION B
[52 marks]

Answer four questions only from this section with at least one question from each part.

All questions carry equal marks.

PART I
PURE MATHEMATICS

9. (a) If $(p + 1)x^2 + 4px + (2p + 3) = 0$ has equal roots, find the integral value of p .

(b) Solve for x and y in the equations: $\log(x - 1) + 2 \log y = 2 \log 3$;
 $\log x + \log y = \log 6$.

10. (a) Differentiate $y = \frac{3x}{1 + x^2}$ with respect to x .

(b) Find the equation of the circle that passes through (2, 3), (4, 2) and (1, 11).

11. When the terms of a Geometric Progression (G.P.) with common ratio $r = 2$ is added to the corresponding terms of an Arithmetic Progression (A.P.), a new sequence is formed. If the first terms of the G.P. and A.P. are the same and the first three terms of the new sequence are 3, 7 and 11 respectively, find the n^{th} term of the new sequence.

PART II

STATISTICS AND PROBABILITY

12. (a) The probabilities that Golu, Kofi and Barry will win a competition are $\frac{1}{3}$, $\frac{2}{5}$ and $\frac{1}{2}$ respectively. Find the probability that **only two** of them wins the competition.

- (b) Ten eggs are picked successively **with** replacement from a lot containing 10 % defective eggs. Find the probability that **at least two** are defective.

13. The marks awarded by three examiners are given in the table:

Candidate	A	B	C	D	E	F	G	H	I	J
Examiner I	90	88	71	65	32	72	70	41	38	14
Examiner II	89	92	70	68	35	66	72	39	40	16
Examiner III	88	89	71	67	36	70	69	38	39	15

- (a) Calculate the Spearman's rank correlation coefficient of the marks awarded by:
- Examiners I and II;
 - Examiners I and III;
 - Examiners II and III.

- (b) Use your results in (a) to determine which of the examiners agree most

VECTORS AND MECHANICS

14. The ends **X** and **Y** of an inextensible string 27 m long are fixed at two points on the same horizontal line which are 20 m apart. A particle of mass 7.5 kg is suspended from a point **P** on the string 12 m from **X**.
- (a) Illustrate this information in a diagram.
- (b) Calculate, correct to two decimal places, $\angle YXP$ and $\angle XYP$.
- (c) Find, correct to the nearest hundredth, the magnitudes of the tensions in the string.
[Take $g = 10 \text{ m s}^{-2}$]
15. A particle **P** moves in a plane such that at time t seconds, its velocity, $\mathbf{v} = (2t\mathbf{i} - t^3\mathbf{j}) \text{ m s}^{-1}$.
- (a) Find, when $t = 2$, the magnitude of the:
- (i) velocity of **P**.
- (ii) acceleration of **P**.
- (b) Given that **P** is at the point with position vector $(3\mathbf{i} + 2\mathbf{j})$ when $t = 1$, find the position vector of **P** when $t = 2$.

END OF PAPER